Does nature convert mass into energy?

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First I provide some history of how the equation $E = mc^2$ arose, establish what "mass" means in the context of this relation, and present some aspects of how the relation can be understood. Then I address the question, Does $E = mc^2$ mean that one can "convert mass into energy" and vice versa?
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I. INTRODUCTION

The relation $E = mc^2$ has spawned an immense literature and some contentious issues. I address one such issue.

But first, let me note that I use $E = mc^2$ (in literally this form) only in a generic sense. This form is, after all, what appears on T-shirts and in cartoons. When I write rigorous physics, the symbol m will always be accompanied by a well-defined identifying subscript, and the symbol E will often have a subscript too.

Section II provides background: some history of how the relation $E = mc^2$ arose, what the word "mass" means in the context of that equation, and some aspects of how the relation can be understood. In Sec. III, I refine the question in the title as "Does the equation $E = mc^2$ mean that we can 'convert mass into energy' and vice versa?" and provide a response. Section IV summarizes the paper with a list of six key ideas.

II. BACKGROUND

A. Increment form

In September 1905, Albert Einstein submitted a short paper[1] whose title was the question, "Does the inertia [Trägheit] of a body depend on its energy content?" Einstein considered a body that emits two bursts of plane electromagnetic waves. In the body's initial rest frame, the bursts emerge back to back and have equal energy. Consequently, the body remains at rest in that frame. In his seminal paper on special relativity,[2] Einstein had derived the Lorentz transformation for the energy of such a burst of plane waves. Thus, he could evaluate the energies of the two bursts in a frame in which the body moves with constant nonzero velocity. After defining the kinetic energy of the body and specializing to the case where the body's speed is much less than c, Einstein found that the body's kinetic energy had decreased (despite the constant velocity) and deduced the relation

$$\Delta m_0 = \Delta E_0 / c^2 \tag{1}$$

(in my notation). Here E_0 denotes the body's rest energy, its energy as evaluated in its rest frame. The symbol m_0 denotes the body's rest mass, the inertia that the body exhibits when it is accelerated from rest.[3]

Einstein used the word "mass" (die Masse) to denote the coefficient of $v^2/2$ in the expression for kinetic energy, and so his "mass" represented inertia. Einstein's major conclusion was that "The mass [that is, the inertia] of a body is a measure of its energy content; if the energy changes by ΔE_0, then the mass changes in the same sense by $\Delta E_0/c^2$ [in my notation]." He also wrote, "If the theory corresponds to the facts, then radiation carries inertia [Trägheit] between the emitting and absorbing bodies."

For the sake of both simplicity and brevity, let me omit all gravitational considerations in this paper. Then, in the context of the equation $E = mc^2$, the letter m and word "mass" denote "inertia."[4]

Inertia and energy are always *attributes* of something, namely, of fields and particles.[5] To fields and particles, physics assigns a different ontological status. (In classical physics, for example, an electron and a proton are immutable "things." Electromagnetic fields have an objective existence in space.) The increment equation, Eq. (1), relates changes in two attributes. For a given body, Eq. (1) states that if one attribute increases, so does the other. The changes are concomitant and in the same sense.

B. From increment form to $E = mc^2$

In 1907, Einstein wrote a paper entitled "On the inertia of energy as required by the principle of relativity."[6] Near the end of the paper Einstein returned to a result that he had derived in his initial paper[2] on relativity theory: how the kinetic energy of an electron, say, depends on its speed and its rest mass. Einstein had derived the result by calculating the work done by an electrostatic field acting over a distance. He found the expression (in my notation)

$$\text{kinetic energy} = m_0 c^2 \left[\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right]. \tag{2}$$

Once this result was established, Einstein dropped the electric charge and used the expression for a neutral body as well.

Einstein remarked that the energy E of the moving particle is the kinetic energy plus a constant: $E = \text{kinetic energy} + \text{const}$. Then he went on: "While in classical [prerelativistic] mechanics it is most convenient to let the arbitrary constant in this equation vanish, in relativistic mechanics one gets the simplest expression for E if one chooses the zero point of energy so that the energy E_0 for the mass point at rest is set to $m_0 c^2$ [in my notation]. Then one finds[7]

$$E = m_0 c^2 \frac{1}{\sqrt{1 - v^2/c^2}} \text{ .''} \tag{3}$$

I find it remarkable that Einstein gave no physical argument for the relation

$$E_0 = m_0 c^2. \tag{4}$$

Rather, he availed himself of the arbitrariness in the zero point of energy and chose the expression that made the energy E simplest. Minkowski's introduction of four-dimensional space-time lay in the future, and so Einstein could get no guidance from four-vectors. Radioactivity was known but not well understood, and the discovery of the positron lay decades in the future. In choosing a zero point so that inertia and energy (for a body at rest) would be strictly proportional (and not merely a linear function of each other), Einstein again displayed superb intuition.

Today, our knowledge that charged particles can be created and annihilated (in pairs) enables one to provide a physically grounded derivation of Eq. (4). An elementary yet compelling derivation is provided in Refs. 8 and 9.

In a footnote in the same paper, Einstein wrote, "It is to be noted that the simplifying stipulation $m_0c^2 = E_0$ [in my notation] is simultaneously the expression of the principle of the equivalence [Äquivalenz] of mass and energy …."[7] As the editors of Einstein's collected papers remark,[10] this footnote is the first time that Einstein wrote of an "equivalence."

Just what did Einstein mean by "equivalence"? The Appendix gives my historical evidence; here I state only my conclusion. All things considered, I think it fair to say that, for Einstein in 1907, the "equivalence of mass and energy" meant a numerical proportionality between the two quantities. Inertia and energy remained distinct—though profoundly related—concepts. No subtle intrinsic identity of the two notions was proposed or intended.

C. Proportionality or identity?

Did Einstein's view of the relation between inertia and energy change with time? Certainly. As the years went by, more sophisticated ways of expressing special relativity theory emerged. Hermann Minkowski's introduction of four-dimensional space-time in 1908 led to a four-vector for energy and momentum and to a tensor that incorporated energy density, energy flux, and momentum density in a single mathematical quantity. The new mathematics revealed new relations between inertia and energy[11] and Einstein's view evolved.

Perhaps Einstein's last written statement about $E = mc^2$ came in his "Autobiographical Notes," commenced when he was age 67 and hence started between March 1946 and a year later. Writing of the insights provided by the special theory, he said, "The laws of the conservation of momentum and the conservation of energy were merged into a single law. The inertial mass of a closed system is identical with its energy, so that [inertial] mass is eliminated as an independent concept."[12]

What exactly did Einstein mean by "identical"? Perhaps the following. If the energy E of the closed system is greater than the magnitude of the momentum (times c) in any inertial frame, then we can transform the energy-momentum four-vector to a frame where the total three-dimensional momentum is zero. In the zero-momentum frame, the four components are given by the set $\{\mathbf{0}, E_0\}$, where E_0 denotes the rest energy. Now make a Lorentz transformation from the zero-momentum frame to a frame moving with the arbitrary velocity $(-\mathbf{v})$ relative to it. In the new frame, the four-vector is given by $\{E_0\gamma\mathbf{v}/c, E_0\gamma\}$, where $\gamma \equiv 1/\sqrt{1 - v^2/c^2}$. In the expression for linear momentum, the rest energy (divided by c^2) plays the role of rest mass. There is no need to introduce separately the notion of rest mass. Einstein described this calculation (in a slightly different form) already in an unpublished review[13] of relativity theory, written primarily in 1912.

Nonetheless, it seems that Einstein could hold a range of views simultaneously. As I evaluate the issue of interpretation, two views about the relation between inertia and energy are tenable.

(1) *Proportionality*. Inertia and energy are conceptually distinct and have separate operational definitions. Invariably, a change in inertia accompanies a change in energy. Provided we adopt the natural zeroes for both inertia and energy, their linear relation becomes the proportionality expressed by $E_0 = m_0c^2$ (or by the version with relativistic mass).

(2) *Identity*. In some intrinsic sense, inertia and energy are identical. Typically, this enigmatic statement means that wherever the notion of inertia is needed, one can replace inertia by rest energy (divided by c^2). The replacement is motivated (or even generated) by natural relations in the four-dimensional formulation of special relativity.[14]

Another way to state the "identity" view is this: a single parameter suffices to specify an object's rest mass and rest energy. For example, quantum field theory introduces just one parameter for a particle, its Compton wavelength λ_C. Then $m_0 = h/c\lambda_C$ and $E_0 = hc/\lambda_C$.

D. Inertia as a dipstick

Prior to concluding this section, I offer an analogy that instructors may find useful. Before I go on a long trip, I check the oil level in my car. I pull out the dipstick, wipe it off, slide it back into the engine, pull it out again, and finally read off the oil level. There is no need to disassemble the engine.

How could a physicist determine the energy content of a specific isotope, ^{62}Ni, for example, without "disassembling" the nucleus into internal kinetic energy, electrostatic potential energy, and so on? First measure the isotope's rest mass (for example, by magnetic deflection at low speed); then use the relation $E_0 = m_0c^2$ to evaluate the rest energy. Inertia provides a dipstick for energy content. Moreover, this assessment of energy content includes the energy that would be released if the nucleons were annihilated (or if their constituent quarks were annihilated).

III. CONVERTING MASS INTO ENERGY AND VICE VERSA

Q. Does the equation $E = mc^2$ mean that one can "convert mass into energy" and vice versa?

A. Not really, but the issue is complex, and eminent physicists have used the phrase or variants of it.

Let me begin a detailed response by listing five reactions that have been useful in discussing the question:

$$p + {}^7\text{Li} \to \alpha + \alpha, \tag{5}$$

$$2\text{H}_2 + \text{O}_2 \to 2\text{H}_2\text{O}, \tag{6}$$

$$\gamma + N \to N + e^- + e^+, \tag{7}$$

$$e^- + e^+ \to 2\gamma, \tag{8}$$

$$p + p \rightarrow p + p + \pi^0. \tag{9}$$

All of these reactions have been observed in the lab, and each is qualitatively different from the others in one respect or another. In the first reaction, a proton as a projectile interacts with a lithium nucleus (at rest), and two alpha particles emerge. Reaction (6) describes the combustion of two dilute gases to produce water vapor. In reaction (7) a gamma ray interacts with a heavy nucleus N (at rest) and produces an electron-positron pair. The two-photon decay of positronium is shown in reaction (8). Finally, an energetic proton interacts with another proton (at rest) and produces a neutral pion.

Conservation of energy and momentum suffice to analyze the kinematics of all these reactions. Of course, one must use relativistically correct expressions for both energy and momentum. If an entity can be at rest in a realizable inertial frame, then its energy is given by Einstein's expression, Eq. (3). Energy and momentum may be combined into an energy-momentum four-vector, and such vectors immensely simplify a calculation of the thresholds for reactions (7) and (9).

No competent physicist makes a mistake in calculating the threshold or other aspects of the kinematics of these reactions. At issue is how the process is described to students and to the public. In being faithful to the physics, should one consider changes in mass as a *source* of an "energy release" or as a *concomitant* of such a release? And what role should the notion of rest energy play?[15]

A. When particles are conserved

The number of protons, neutrons, and electrons is conserved in reactions (5) and (6). Those particles, taken as fundamental particles, are merely rearranged.

For the case of combustion, the initial three molecules have (on average) a total kinetic energy (associated with their individual center of mass motions) equal to $3(\frac{3}{2}kT)$, where T is the ambient temperature. At room temperature, the total kinetic energy amounts to 0.1 eV. The reaction products share a kinetic energy of 5 eV. How do we explain a 50-fold increase in the kinetic energy? We say that the rearrangement has changed the potential energy of the intramolecular electrostatic forces and has altered the internal kinetic energy of the electronic and nuclear motions. The total internal molecular energy has decreased, and that suffices to account for the increase in the kinetic energy of the center of mass motions.

Ought we not say something closely analogous in the case of reaction (5)? When Cockcroft and Walton performed their classic experiment[16] in 1932, they found disintegration of lithium already when the incident proton had a kinetic energy of only 125 keV. The two alpha particles shared a kinetic energy of 17 MeV. How do we explain the 140-fold increase in the kinetic energy (of the center of mass motions)? The two protons and two neutrons in an alpha particle (the first doubly magic nucleus) are especially tightly bound. Rearrangement of the nucleons changes the intranuclear potential energy (which arises from nuclear and electrostatic forces) and the intranuclear kinetic energy of the nucleons. Spatial separation of the two alpha particles, which repel each other, generates a further change in the system's electrostatic potential energy. The total internal energy of the nuclei has decreased, and this decrease suffices to account for the increase in the kinetic energy of the center of mass motions. The parallel with combustion is strikingly close.

If we keep our focus on energy, then each molecule or nucleus has an energy E that we can write as

$$E = E_0/\sqrt{1 - v^2/c^2} = E_0 + K, \tag{10}$$

where K denotes the kinetic energy of the center of mass motion. Conservation of energy takes the form

$$\sum E_0 + \sum K = \text{const}. \tag{11}$$

At any instant of time, the sums go over all the presently existing participants in the reaction.

Next, evaluate the expressions in Eq. (11) for both the initial and final states and take the difference. The outcome is

$$\sum_{\text{initial}} E_0 - \sum_{\text{final}} E_0 = \Delta K_{\text{total}}. \tag{12}$$

The sums go over the initial and final participants in the reaction. The difference between the initial and final rest energies equals the change in the total kinetic energy (of the center of mass motions). In short, changes in the rest energies are responsible for the dramatic increase in the total kinetic energy. (Because the number of protons, neutrons, and electrons has remained constant, the changes in rest energies here arise strictly from changes in the internal potential and kinetic energies.)

Changes in rest mass have not yet been introduced. We can, however, turn now to "the equivalence of mass and energy," taken to mean that rest mass and rest energy are universally proportional to each other. (Or we can recall that "inertia provides a dipstick for energy content.") The relation $E_0 = m_0 c^2$ implies that the decrease in summed rest energies was accompanied by a decrease in summed rest masses. This change in inertia is a concomitant of the change in rest energies.

Mass spectroscopy provides values of inertial masses and hence, at low speed, values of rest masses. Cockcroft and Walton compared the difference of the summed rest masses (times c^2) with the observed change in total kinetic energy. The comparison provided brilliant support[17] for the proposition that $\Delta E_0 = \Delta m_0 c^2$. Their experiment, however, does not warrant the inference that "inertia has been converted to energy."[18] The energy has been there all along. Internal energy (of various sorts) has been converted to the kinetic energy of center of mass motions. Therefore the sum of rest energies is smaller, and—merely as a concomitant—the sum of rest masses is smaller. The experiment's true import is that changes in rest energy and rest mass are proportional.[19,20]

B. Annihilation and creation

The positron was first observed in 1932, and physicists learned about annihilation and creation. A photon is annihilated in the field of a heavy nucleus, say, and an electron-positron pair is created, as displayed in reaction (7). An atom of positronium decays and produces two or three gamma rays, as shown for two photons in reaction (8).

Reactions in which electromagnetic radiation is annihilated and matter is created (or vice versa) are constrained by conservation of total energy and total momentum. Energy can be expressed in terms of rest energies, kinetic energies, and the energy E_γ of electromagnetic radiation. An equation like Eq. (11) needs to be augmented with E_γ terms:

$$\sum E_0 + \sum K + \sum E_\gamma = \text{const.} \quad (13)$$

Otherwise, nothing new arises.

The production of an electron-positron pair in reaction (7) requires a certain minimum energy for the incident gamma ray. The threshold energy can be calculated in terms of the rest energies for the two leptons and the heavy nucleus. Only after this calculation has been done is there any reason to turn to the equation $E_0 = m_0 c^2$ and express the threshold in terms of rest masses.

Given the physics described in the preceding three paragraphs, is pair production an example of "converting energy to mass"?[21] No, for the following reasons.

First of all, a photon is not just "energy." There is no such thing as "pure energy." Rather, the photon is the particle of the electromagnetic field. It does possess the *attribute* of energy. The electron and positron possess the property of rest mass (as well as energy), but they are not "mass" (in the sense of "*being* inertia"). Perhaps, because they are "matter," the electron and positron are mass in the sense of "lump of stuff." That sense of mass, however, is different from the sense in which the word mass is used in the equation $E = mc^2$.

Next, we should ask, "Because energy is conserved, how could energy be converted into something else or vice versa?" If "conversion" is just a metaphor, then we ought to ask whether it is misleading. If so, then we should consider dropping it.

In summary, the notion of rest energy plus conservation of energy suffice to quantify the energetics of pair creation. Moreover, they provide a route that is generally applicable and that is logically economical and impeccable. The profound connection between inertia and energy may be used separately and subsequently.

In a significant sense, the annihilation of the electron and positron in positronium is like a movie of pair creation run in reverse. For the topic of $E = mc^2$, there is nothing intrinsically new. After noting that the annihilation does *not* "convert mass into energy," we may go on.

The last of the five reactions features the production of a new particle while the original particles remain on the scene (though with altered energies and momenta). As always, conservation of energy and momentum imposes constraints on the process. Energetically, the final state includes the rest energy of the pion and—when observed from the lab frame—some kinetic energy because conservation of momentum requires some motion even at threshold. Thus the initial state must have had more kinetic energy than the final state. Creation of the pion was accompanied by a reduction in total kinetic energy [as Eq. (13) indicates].

Does reaction (9) constitute transformation of energy into mass? No. Kinetic energy has been transformed into rest energy, but that transformation is merely analogous to transforming kinetic energy to potential energy (as in the motion of an object attached to a horizontal spring).

C. Is mass conserved?

In the five reactions that we have just examined, energy is conserved, but is the sum of the individual masses also conserved? The answer depends on which quantitative definition for mass one adopts. If one adopts relativistic mass,[22] then the relation $E = m_{\rm rel} c^2$ plus conservation of energy imply that the sum of the (relativistic) masses is conserved. The other alternative has two pieces: use the rest mass when the particle can be at rest in some realizable inertial frame; assign a zero mass to a photon. Given this choice of definition, the sum of the final masses differs from the sum of the initial masses in all five reactions.

The choice between relativistic mass and rest mass is irrelevant to the central question of this section. The key elements are rest energy, conservation of energy, and understanding that the mass in $E = mc^2$ means inertia.

D. Alternative views

In his second book on the concept of mass, Max Jammer wrote that there are at least two schools of thought on what the relation $E = mc^2$ implies. As he noted, "According to one interpretation the relation expresses the convertibility of mass into energy or inversely of energy into mass, with one entity being annihilated and the other being created."[23]

A brief list of authors who write of "conversion" generates a distinguished collection: Max von Laue,[24] Wolfgang Pauli,[25] Edwin F. Taylor and John A. Wheeler,[26] Julian Schwinger,[27] and even Einstein.[28]

I have looked in vain for the explicit logic of the "conversion" point of view. What seems to be operating is a process of substitution followed, sometimes, by misinterpretation. Here are the steps. First, for any given reaction, conservation of energy leads to the relation

$$\Delta \sum E_0 = -\Delta\Big(\sum K + \sum E_\gamma\Big). \quad (14)$$

The Δ symbol indicates the difference between the final and initial states. The sums go over all participants for which a rest energy and kinetic energy are defined, and the energy of electromagnetic radiation is entered separately. Next, the equation $E_0 = m_0 c^2$ is invoked:

$$\Delta \sum E_0 = \Big(\Delta \sum m_0\Big) \times c^2. \quad (15)$$

Upon substituting for the rest energies in Eq. (14), we find

$$\Big(\Delta \sum m_0\Big) \times c^2 = -\Delta\Big(\sum K + \sum E_\gamma\Big). \quad (16)$$

Equation (16), which was derived by substitution and is certainly correct, seems to provide the justification for saying that mass can be converted to energy and vice versa. But what is this mass?

In the conversion interpretation, there is often a whiff of mass as "lump of stuff" that makes the phrase appealing. Yet Einstein was clear that mass means inertia in his famous relation. I wonder how appealing conversion would sound if it were phrased as "convert inertia into energy and vice versa"?

The essence of the conversion issue seems to lie in a loose use of language.[29]

Perhaps the best way to gain clarity is to draw the analogy with energy conservation in nonrelativistic physics. Imagine that a piece of aluminum slides without friction on a horizontal air track and is tethered at each end by a spring. Once set into motion, the object oscillates back and forth. The sum of potential energy and kinetic energy is conserved. We speak freely and correctly of converting potential energy to kinetic energy and vice versa.

For the reactions considered in this section, Eq. (13) provides the analogous conservation law for energy. If we are careful in the language that we use, we will say that an increase in the second and third sums is accompanied by a decrease in the first sum. That is to say, rest energy is converted to kinetic energy and/or the energy of electromagnetic radiation and vice versa.

Near the end of Sec. II, I noted that the relation $E_0 = m_0c^2$ may be described as expressing a proportionality or an identity. Either way, what changes—according to Eq. (13)—when total kinetic energy and electromagnetic energy change is *rest energy*. It is not inertia *per se*. In the "proportionality" view, the change in inertia is a concomitant of the change in rest energy. In the "identity" view, inertia shows up elsewhere: in the momentum equations, not in the energy equation.

As this paper has illustrated, the equation $E_0 = m_0c^2$ can be used at various stages in a calculation to replace a rest energy E_0 by an inertia m_0 (times c^2). This diversity can be a source of confusion. There is no rule for proper procedure; one just has to be alert to what's going on.

In conclusion, if we are careful in the language that we use and are mindful of the substitutions that have been made, we can avoid implying conversions that are not indicated by the equations when taken literally and in their primary form.

IV. PERSPECTIVE

Here I summarize the key ideas in this paper.

(1) Ontological status makes a difference. Particles and fields exist as "things;" inertia and energy exist as attributes of things.
(2) In the context of $E = mc^2$, the letter m and the word mass denote "inertial mass," that is, inertia.
(3) At the most fundamental level, conservation of energy includes rest energy, which is conceptually distinct from rest mass.
(4) Of course, the relation $E_0 = m_0c^2$ declares that rest energy and rest mass are universally proportional. Inertia, a dynamically measurable quantity, provides a dipstick for extractable energy content.
(5) One may regard inertia and energy as "identical" in the sense that a single parameter suffices to determine both the rest mass and the rest energy of a particle or closed system.
(6) One can analyze reactions (including nuclear fission and the creation and annihilation of particles) by using conservation of energy and momentum alone. There is no need to speak of "converting" mass to energy or vice versa.

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APPENDIX: HISTORICAL EVIDENCE

Here I give the historical evidence for my conclusion about what Einstein meant, in 1907, by the phrase "the equivalence of mass and energy."

The English word "equivalence" may have connotations that Einstein did not intend when he used the German word "Äquivalenz." So, a first step is to consult an authoritative German dictionary and find out what meanings it offers.

Der Grosse Brockhaus defines "Äquivalenz" as "equal in value" (Wertgleichheit) and "equivalent" (Gleichgeltung).[30] A survey of four other German-to-German dictionaries, including a *Brockhaus* published in 1898, produced no significantly different definitions. The definition as "equal in value" suggests that Einstein meant a numerical equality between energy and inertial mass (times c^2). Nonetheless, because the English word "equivalent" has a spectrum of meanings, resort to the dictionary leaves us short of a definitive understanding. What guidance can we gain from Einstein's other statements about the "equivalence"?

In 1913, while writing about the nascent theory of general relativity, Einstein began a sentence with the words, "On the one hand, the proportionality of energy and inertial mass that the usual [special] theory of relativity produces, …."[31] In a paper on the foundations of general relativity, submitted in 1914, Einstein wrote, "Finally, I return once again to the law of the identity of inertial and gravitational mass and to the connection [Zusammenhang] between mass and energy."[32] Although Einstein asserted an identity in one comparison, he claimed only a connection between mass and energy.

Moreover, all experience had shown that a body's inertia is a positive quantity. A body's energy, as Einstein repeatedly noted, has an arbitrary zero. The energy could be negative as well as positive. I find it difficult to imagine that Einstein, in 1907, would have considered two such disparate quantities to be intrinsically identical.

These are the major historical facts and logical considerations that led me to the conclusion stated in the main text: in 1907, "equivalence" meant a numerical proportionality between inertia and energy.

A comment, however, is in order. For a moment, let me deviate from my decision to omit gravitational considerations. An aspect of what Einstein had to say about the relation of inertial and gravitational mass is instructive. In comparing inertial mass and passive gravitational mass, Einstein wrote of their proportionality (Proportionalität),[33] identity (Identität), "physical identity" (physikalische Wesengleichheit), equivalence (Äquivalenz), equality (Gleichheit), and equality (proportionality) [Gleichheit (Proportionalität)]. I have given these descriptions in chronological order (or, when they occur in a single paper, in order of appearance). Do they reflect a progression in Einstein's views? I doubt it. For example, four descriptions—Proportionalität, Identität, physikalische Wesengleichheit, and Äquivalenz—are used in a single paper. Moreover, quotations from later publications would show an irregular alternation of several of the terms together with the addition of "agreement" (Übereinstimmung). Although Einstein certainly qualified as a major philosopher of science, his casual use of language does not measure up to the standard of the profession today. One should bear that characteristic in mind when reading isolated pieces of Einstein's voluminous writings.

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[1]*The Collected Papers of Albert Einstein*, edited by John Stachel, David C. Cassidy, Jürgen Renn, and Robert Schulmann (Princeton U. P., Princeton, NJ, 1989), Vol. 2, "The Swiss Years: Writings, 1900–1909," pp. 311–314.

[2]Reference 1, pp. 275–306.

[3]The word *inertia* denotes an object's (or a system's) reluctance to undergo a change in velocity. Inertia is distinct from momentum. For example, a golf ball exhibits a "reluctance to undergo a change in velocity" even when at rest on the tee. That's why one has to whack it with a golf club to send it down the fairway.

[4]Inertia can be given an operational definition. For example, if a mass spectrometer uses a "velocity filter" in the sense that only particles of a specific velocity make it through a region of crossed electric and mag-

netic fields and then magnetic deflection at low speed, the outcome is a measurement of the particle's rest mass, that is, its inertia when it is accelerated from rest. Thus inertia can be given an operational meaning and one that is independent of the notion of energy.

[5] In the most fundamental physics—quantum field theory—the expression for energy is derived from a Lagrangian for fields and is expressed in terms of those fields. Thus, I maintain, energy is a property—an attribute—of fields (and particles). To be sure, other views exist. In his *Concepts of Mass in Classical and Modern Physics* (Harvard U. P., Cambridge, MA, 1961), Max Jammer writes of inertial mass and energy as "the same physical substratum" (p. 188) and writes of the "reification of energy" (p. 173).

[6] Reference 1, pp. 413–427.

[7] Reference 1, p. 425.

[8] Ralph Baierlein, "Teaching $E=mc^2$: An exploration of some issues," Phys. Teach. **29**, 170–175 (1991). The derivation uses the notion of relativistic mass, but the limit of vanishing speed yields Eq. (4).

[9] Ralph Baierlein, *Newton to Einstein: The Trail of Light* (Cambridge U. P., New York, 1992), pp. 236–269, and pp. 319–325.

[10] Reference 1, p. 428.

[11] For example, Wolfgang Pauli, *Theory of Relativity* (Pergamon, New York, 1958), p. 125 (in conjunction with note 146 on p. 86).

[12] Albert Einstein, "Autobiographical Notes," in *Albert Einstein: Philosopher-Scientist*, edited by Paul Arthur Schilpp (Tudor, New York, 1949), p. 60.

[13] *The Collected Papers of Albert Einstein*, edited by Martin J. Klein, A. J. Kox, Jürgen Renn, and Robert Schulmann (Princeton U. P., Princeton, NJ, 1995), Vol. 4, "The Swiss Years: Writings, 1912–1914," pp. 97–98.

[14] A calculation that one person views as showing that inertia and energy are necessarily proportional another person may view as indicating that inertia and energy are identical. The difference in viewpoints may not be as great as it seems. The two camps would agree, I believe, that—most fundamentally—the connection between inertia and energy follows from the stress-energy tensor.

[15] Rest energies figure prominently in the discussion that follows, and so a detailed example is in order. Consider a hydrogen atom whose center of mass is at rest. The atom's rest energy consists of the electrostatic potential energy (of the electron-proton interaction), the kinetic energy of motions relative to the center of mass (primarily the electron's kinetic energy), and the rest energies of the electron and proton. According to the standard model, the electron's rest energy cannot be dissected into distinct contributions. The proton's rest energy could be described in terms of the rest energies of the constituent quarks, their motion (internal to the proton), and their interaction energy.

[16] J. D. Cockcroft and E. T. S. Walton, "Experiments with high velocity positive ions. II. The disintegration of elements by high velocity protons," Proc. R. Soc. London, Ser. A **137**, 229–242 (1932).

[17] Roger H. Stuewer, "Mass-energy and the neutron in the early thirties," Sci. Context **6**, 195–238 (1993). Stuewer points out that Cockcroft and Walton intended their comparison to support their inferred value of the kinetic energy, not to test Einstein's relation (whose validity they assumed). Kenneth T. Bainbridge published a test based on their experiment in "The equivalence of mass and energy," Phys. Rev. **44**, 123 (1933), and found agreement within the probable error, which was approximately 3%.

[18] "This Month in Physics History: Energy and mass are equivalent," APS News **14**(4), 2 (2005). The anonymous author writes, "Meanwhile, in Cambridge, England, the reverse process was seen in 1932: the conversion of mass into energy. With their apparatus, John Cockcroft and E. T. S. Walton …." Such statements are common in books and articles; in Sec. III D, I cite some other instances.

[19] Simon Rainville *et al.*, "A direct test of $E=mc^2$," Nature (London) **438**, 1096–1097 (2005). Despite the title, a careful reading shows that the authors tested the increment equation $\Delta E_0=\Delta m_0 c^2$. Their impressive accuracy is the best yet achieved: a few parts in ten million.

[20] Because the number of protons and neutrons remains constant in nuclear fission, one can understand the energy release in terms of changes in potential energy, kinetic energy, and the energy of electromagnetic radiation. There is no need to invoke $E=mc^2$. The analysis for nuclear fission is developed in an elementary fashion in Ref. 9, pp. 248–252 and 256–260.

[21] Reference 18. The author writes, "…Iréne and Frédéric Joliot-Curie obtained direct photographic evidence of the conversion of energy into mass."

[22] The relativistic mass $m_{\rm rel}$ is defined as the proportionality factor between momentum $\mathbf{p}$ and velocity $\mathbf{v}$: $\mathbf{p}=m_{\rm rel}\mathbf{v}$. The relativistic mass may depend on the object's speed.

[23] Max Jammer, *Concepts of Mass in Contemporary Physics and Philosophy* (Princeton U. P., Princeton, NJ, 2000), pp. 77–82.

[24] Max von Laue, "Inertia and energy," in *Albert Einstein: Philosopher-Scientist*, edited by Paul Arthur Schilpp (Tudor, New York, 1949), pp. 503–533.

[25] Reference 11, p. 217.

[26] Edwin F. Taylor and John Archibald Wheeler, *Spacetime Physics*, 2nd ed. (Freeman, New York, 1992), pp. 237, 243, and 248–249.

[27] Julian Schwinger, *Einstein's Legacy: The Unity of Space and Time* (Scientific American Books, New York, 1986), pp. 98–111.

[28] Reference 1, p. 465. *The New Quotable Einstein*, edited by Alice Calaprice (Princeton U. P., Princeton, NJ, 2005), p. 245. Also at ⟨www.aip.org/history/einstein/voice1.htm⟩.

[29] Reference 27 provides an illuminating example. Schwinger starts with "rest energy" (p. 98), slips to "rest-mass energy" (p. 99), and then converts "rest mass" to kinetic energy (pp. 105, 108, and 110–111).

[30] *Der Grosse Brockhaus, Brockhaus' Konversations-Lexikon* (Eberhard Brockhaus, Wiesbaden, Germany, 1952), Vol. 1, p. 352.

[31] Reference 13, p. 482.

[32] Reference 13, p. 575.

[33] The seven different characterizations that I display in this paragraph come from Ref. 13, pp. 304, 305 (twice), 322, 488, 489, and 585, respectively.