How Does the Potential Energy of a Rising Helium-Filled Balloon Change?

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When I recently posed the title question to my students and presented them with my answer, their strong initial skepticism led to one of the best class discussions I can remember. Suddenly my students were more motivated to understand what makes a force conservative, and the relationship between conservative forces and their associated potential-energy functions. The surprising answer I gave was that as a helium-filled balloon rises through the air, its total potential energy *decreases*.

This is because there are, in fact, two conservative forces that act on the balloon. These are gravity and the buoyant force. The *total* potential energy associated with these two conservative forces may be seen to decrease as the balloon rises. I have examined numerous introductory physics books without finding any discussion of a buoyant force being conservative and therefore having an associated potential energy. In this note I discuss why the buoyant force is conservative and how this fact implies that when a helium-filled balloon rises through the air, or for that matter when an ice cube or a gas bubble rises through water, the total change in its potential energy is negative.

Imagine a simple experiment conducted in the classroom or laboratory. A balloon is filled with a sufficient amount of helium so that it will rise through the air with constant velocity. Using Archimedes' principle, we can write the buoyant force as:

$$F_b = \rho V g \,, \tag{1}$$

where ρ is the density of the air in the room and V is the balloon's volume. For such a balloon, this force is constant and directed vertically upward. To establish that the force is conservative, we need only compare it to the gravitational force

$$F_g = -\, mg \,, \tag{2}$$

which has the associated potential energy function

$$U_g = mgy \,, \tag{3}$$

where y is the distance above the reference level (taken to be at the ground). In exactly the same way, we may write the potential energy associated with the buoyant force as:

$$U_b = -\rho V g y \,. \tag{4}$$

In both cases, we have, as required,[1,2]

$$F(y) = -dU(y)/dy \,. \tag{5}$$

The total potential energy associated with the two conservative forces is

$$U = (mg - \rho V g)y. \tag{6}$$

To establish how this function varies with height y, we must compare the magnitudes of the two terms in the parentheses. If only gravity and the buoyant force were acting on the balloon, then motion with constant velocity would imply that $mg = \rho V g$. In that (impossible) situation, we would have the trivial case $U(y) = 0$. But, of course, as the balloon rises through the air it experiences a downward force of air resistance, so for constant velocity:

$$\rho V g - mg - f = 0, \tag{7}$$

where f is the magnitude of the (nonconservative) resisting force. From this equation it is clear that $\rho Vg > mg$. This being the case, we may immediately see from Eq. (6) that as y increases, the total potential energy decreases.

Since my students are quite surprised by this result, I like to have a couple of alternate explanations at hand. Another way of looking at the situation is in terms of the total mechanical energy of the rising balloon. We may write

$$\Delta K + \Delta U = W_{\mathrm{N}}\,, \tag{8}$$

where ΔK, the change in kinetic energy, is equal to zero, and W_{N} is the total work done by nonconservative forces acting on the balloon. The only nonconservative force is f, and the work it does as the balloon rises is negative. Therefore, the total change in potential energy as the balloon rises is again seen to be negative.

And, finally, here is a descriptive argument that even my most skeptical students find convincing. In my class, I like to describe the process of increasing an object's potential energy in terms of "fighting against" a conservative force. In lifting a heavy object, one opposes the force of gravity in increasing its (gravitational) potential energy. Now for the balloon we've been considering, the resultant conservative force is directed upward. So, if we displace the balloon downward, in opposition to that force, we increase its potential energy. Thus, when it is closer to the ground, the balloon has more potential energy than when it is higher.

It is possible to extend the preceding discussion somewhat by relaxing the constraint that the balloon remains close to the ground. In that case, neither the gravitational force nor the buoyant force is constant. Now, the situation is more complex, but if we assume that both forces remain radial (directed toward the center of Earth), then both are still conservative.

In conclusion, I recommend posing the title question of this note to your students when you are covering the concepts of work and energy. I believe that you will find, as I did, that the initial surprise at the answer will motivate the class to think more deeply about conservative forces and their relationship to potential energy.

References

1. See, for example, Raymond Serway, *Physics for Scientists & Engineers,* 4th ed. (Saunders College, Philadelphia, 1996), p. 217.
2. Of course, using the same arguments one employs for the gravitational force, one can show that the work done by F_{b} as the balloon moves is independent of path and that the total work done by F_{b} is zero if the path is closed.