$$F = 4\pi G m \rho \int_0^{\pi/2} (R^3/r^2)\sin\phi d\phi \cos^2\phi$$
$$= GmM/r^2, \qquad (5)$$

where $M = 4\pi R^3\rho/3$ is the mass of the Earth.

The proof has been presented for the case of a homogeneous sphere, but it is easily generalized to the case of a spherical shell, and hence to the case of an arbitrary spherically symmetric mass distribution, by means of an obvious superposition argument.[3]

This method is particularly simple when the test mass is on the Earth's surface, i.e., $r = R$. In this case the limits on x are 0 to $2R\cos\theta$, the limits on θ are from 0 to $\pi/2$, and ϕ is not needed. For certain courses, the present derivation in general might be judged too difficult for students, yet it would be desirable to present more than a mere assertion of the theorem's validity. In these instances, restriction of the presentation to the special case $r = R$, which is perhaps the case of greatest interest, would be an attractive intermediate option.

Also, for $r = R$ the present method is easily generalized to non-inverse-square forces. For the force law, $F_{12} = G'm_1m_2/r_{12}^n$, it is found by the same method[4] that ($n < 3$)

$$F = 2\pi G' m\rho(2R)^{3-n}/(3-n)(5-n),$$

and therefore, the property of "effective mass concentration at the Earth's center" holds only for $n = 2$.

[a]Supported in part by the National Research Council of Canada.

[b]Present address: Cryogenics Division, National Bureau of Standards, Boulder, Colorado 80302.

[1]See, for example, D. Halliday and R. Resnick, *Fundamentals of Physics* (Wiley, New York, 1974), p. 255.

[2]See, for example, R. Murray and G. Cobb, *Physics: Concepts and Consequences* (Prentice-Hall, Englewood Cliffs, NJ, 1970), p. 153.

[3]The method could be generalized to, but is not particularly suited for, the case of a mass inside the Earth, i.e., $r < R$. However, it is easily shown that the force of gravity inside a spherical shell is zero because of cancellation of forces from area elements on opposite sides of the test mass. That argument can be used to supplement the proof here for the case $r < R$. See Ref. 1, p. 273.

[4]The force of gravity becomes infinite for $n \gtrsim 3$.

Work and kinetic energy for an automobile coming to a stop

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A standard problem in introductory mechanics is the following one taken from Halliday and Resnick: "Show from considerations of work and kinetic energy that the minimum stopping distance for a car of mass m moving with speed v along a level road is $v^2/2\mu_s g$, where μ_s is the coefficient of static friction between tires and road."[1] In the solution of this problem one usually refers to the external frictional force on the car and one sets what seems to be the work of this frictional force equal to the change in the kinetic energy of the car. That is, $-f_s d = 0 - \frac{1}{2}mv^2$, $\mu_s mgd = \frac{1}{2}mv^2$, and $d = v^2/2\mu_s g$. However, as is well known, the frictional force between tires and roadway does no work (assuming no slipping). Hence, in the solution of this problem the particle model forces one into the difficult situation of seeming to say that the frictional force does work, when we know that it does no work. How are we to handle this situation as teachers?

In the first place, we must note that the automobile is not a particle. So perhaps one way out of the difficulty is to treat the automobile as a collection of particles and to write

$$W_{\text{ext forces}} + W_{\text{int forces}} = \Delta K.$$

The work done by the brake friction forces shows up in the sum $W_{\text{int forces}}$. Since the external forces do no work we have $W_{\text{int forces}} = \Delta K$. This is all right as far as it goes, but how do we calculate the work of the internal forces and get the minimum stopping distance? We come back to the original solution to the problem.

Since the original solution is correct we must clarify the principle involved in this solution (i.e., clarify it to our students). For a system of particles we have

$$\Sigma \mathbf{F}_{\text{ext}} = M\mathbf{a}_{\text{cm}},$$
$$\int \Sigma \mathbf{F}_{\text{ext}} \cdot d\mathbf{r}_{\text{cm}} = \Delta(\tfrac{1}{2}Mv_{\text{cm}}^2). \qquad (1)$$

Equation (1) looks very much like the work-energy theorem for a particle. However, the term on the left is *not equal to the work of the external forces unless the infinitesimal displacement for each external force equals the infinitesimal center-of-mass displacement,* $d\mathbf{r}_{\text{cm}}$ *(pure translation).* This is what causes the trouble with the automobile stopping problem. The left side of the equation is $-f_s d$, but this is not equal to the work of the frictional force. The work of the frictional force is zero, as previously mentioned. Equation (1) should not really be called a work-energy theorem (except for pure translation).

The important thing is that we be rigorous in our teaching and that we carefully avoid a trap such as the one which is implicit in the wording of the Halliday and Resnick problem[2] originally quoted.

The author wishes to acknowledge a helpful discussion with his colleague William Chinitz on this problem.

[1]D. Halliday and R. Resnick, *Fundamentals of Physics*, revised printing (Wiley, New York, 1970), p. 108a, problem 24(5).

[2]This problem is also handled in a misleading way by other authors. For example, K. Ford, in *Classical and Modern Physics, Vol. 1* (Xerox College, Lexington, MA, 1972), p. 430, problem E10.16, gives the same problem and refers it to his section on "Work and kinetic energy for one-dimensional motion."