system the force on a charge moving with arbitrary velocity $\mathbf{u}$ is given by Lorentz's force $\mathbf{f} = e\mathbf{E} + (e/c)(\mathbf{u} \times \mathbf{H})$. In the moving system S' the force is given by the same equation, $\mathbf{f}' = e\mathbf{E}' + (e/c)(\mathbf{u}' \times \mathbf{H}')$. Using the force transformations derived above, the velocity transformation formulas[1] and the proper choice of the arbitrary velocity $\mathbf{u}$, we may readily obtain the field transformation equations.[3]

[1]A. P. French, *Special Relativity*, The M.I.T. Introductory Physics Series (Norton, New York, 1968).

[2]H. Goldstein, *Classical Mechanics* (Addison-Wesley, Reading, MA, 1953).

[3]J. D. Jackson, *Classical Electrodynamics*, (Wiley, New York, 1962).

Pseudowork-energy principle

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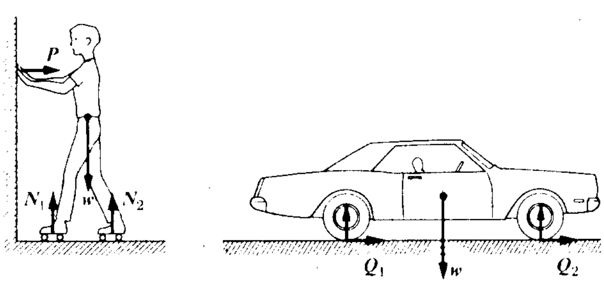
In this note, a theorem is presented which relates the translational energy of the center of mass of an extended body to the net externally applied force. This theorem at first glance looks something like the usual work-energy principle. It, however, can allow one to bypass the difficulties involved with consideration of internal forces, or vibrational and rotational kinetic energies. It is especially helpful as a teaching tool to explain situations in which all the work on nonrigid and/or rotating objects is done by internal forces. We call this the pseudowork-energy principle; it equates the pseudowork to the change in translational kinetic energy of the center of mass.

The so-called work-energy principle[1] (or work-energy theorem[2]) states that *"the work of the resultant force exerted on a particle equals the change in kinetic energy of the particle."* (See, e.g., Ref. 1, p. 95 or Ref. 2, p. 142.) This general principle, a form of the law of conservation of energy, simplifies some otherwise difficult problems in particle dynamics in elementary physics courses. When applied to the dynamics of nonrigid bodies of finite extent, the principle shows that (see, e.g., Ref. 1, p. 105)

$$W = W_e + W_i = \Delta E_k. \tag{1}$$

The total work W of all forces is the sum of the work W_e done by external forces and the work W_i done by internal forces. The total kinetic energy E_k can be regarded as consisting of two parts—kinetic energy due to translation of the center of mass, and kinetic energy of particles relative to the center of mass. This latter part consists of rotational and vibrational kinetic energies. (The vibrational kinetic energy is zero if the body is rigid.)



(a) (b)

7-11 (a) External forces acting on a man who is pushing against a wall. The work of these forces is zero. (b) External forces on an automobile. The work of these forces is zero. In both cases, the work of the internal force is responsible for the increase in kinetic energy.

Fig. 1. Reprinted from F. W. Sears and M. W. Zemansky, *University Physics*, 4th ed. (Addison-Wesley, Reading, MA, 1970) (with permission of the publisher).

This combination of internal and external work with translational, rotational, and vibrational kinetic energies, makes the work-energy principle difficult for beginning students to understand and apply. This is particularly true in cases where although the external forces do no work and thus do not increase the kinetic energy, they are responsible for the change in momentum (since internal forces can not change momentum). Two such cases are illustrated in Fig. 1.

We prove a theorem below which in fact relates the external forces to the translational kinetic energy, and thus takes the mystery out of the apparent paradox of forces which change momentum without changing energy.

It is well known (Ref. 2, p. 189 or Ref. 1, p. 58) that Newton's second and third laws of motion together imply that for an extended object

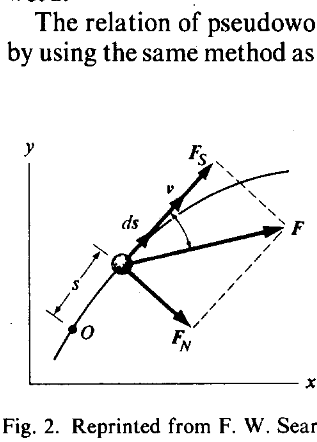
$$\mathbf{F}_{\text{ext}} = M\mathbf{a}_{\text{cm}}. \tag{2}$$

We define a quantity which we call the *pseudowork* on an object, obtained by taking the scalar product of the resultant external force with the translation vector (displacement) of the center of mass of the object.

$$W_{ps} \equiv \text{Pseudowork} \equiv \int \mathbf{F}_{\text{ext}} \cdot d\mathbf{s}_{\text{cm}}. \tag{3}$$

Note that the external forces need not be applied to the center of mass, and the points of application of these external forces need not have the same displacement as the center of mass; this is not true work in the usual sense of the word.

The relation of pseudowork to energy-changes is found by using the same method as used to prove the work-energy



7-1 Path of a particle in the xy-plane.

Fig. 2. Reprinted from F. W. Sears and M. W. Zemansky, *University Physics*, 4th ed. (Addison-Wesley, Reading, MA, 1970) (with permission of the publishers).

principle (Ref. 1, p. 94). With reference to Fig. 2, F_N changes only the direction of the velocity, whereas F_s changes only the magnitude of the velocity. Thus the pseudowork done in going from initial position i to final position f is

$$\begin{aligned}\text{Pseudowork} &= \int_i^f F_s ds_{\text{cm}} = \int_i^f M\left(\frac{dv_{\text{cm}}}{dt}\right) ds_{\text{cm}} \\ &= M\int_i^f \left(\frac{ds_{\text{cm}}}{dt}\right)\left(\frac{dv_{\text{cm}}}{ds_{\text{cm}}}\right) ds_{\text{cm}} \\ &= M\int_i^f v_{\text{cm}}\left(\frac{dv_{\text{cm}}}{ds_{\text{cm}}}\right) ds_{\text{cm}} \\ &= M\int_{v_{i\text{cm}}}^{v_{f\text{cm}}} v_{\text{cm}}\, dv_{\text{cm}} \\ &= \tfrac{1}{2} M v_{f\text{cm}}^2 - \tfrac{1}{2} M v_{i\text{cm}}^2. \qquad (4)\end{aligned}$$

Hence, we have proved the theorem that

$$\int \mathbf{F}_{\text{ext}} \cdot \mathbf{ds}_{\text{cm}} = \Delta(\tfrac{1}{2} M v_{\text{cm}}^2), \qquad (5)$$

or

$$W_{ps} \equiv \text{Pseudowork} = \Delta(\text{KE}_{\text{cm}}), \qquad (5a)$$

i.e., *the pseudowork of the resultant external force exerted on an object equals the change in kinetic energy of translation of the center of mass of the object.* We call this theorem the *pseudowork-energy principle.*

It is interesting to note that a student who does not understand the work energy principle too well will often use pseudowork instead of work for the left-hand side of Eq. (1) and will neglect the rotational and/or vibrational parts of the kinetic energy on the right-hand side of Eq. (1), thus making two compensating errors and inadvertently arriving at the correct principle of Eq. (5). Surely the pseudowork-energy principle has also been arrived at correctly many times in the past by those who did understand both work and energy. However, we know of no reference to it in the literature, and it is surely absent in standard elementary physics texts.

In cases (such as those in Fig. 1) where the external forces are applied at motionless points, and thus do no true work, the change in KE_{cm} is only one part of the total work by the internal forces. Yet, without external forces, the internal forces could not change the momentum and thus could not increase KE_{cm}. The pseudowork-energy principle shows the quantitative relation between the external forces and KE_{cm}. It thus helps clarify the situation in which external forces increase momentum without doing work. Also, since pseudowork involves only external forces and the motion of only one point (the center of mass), it is often much easier to compute than the real work. (See example in Appendix.) Thus the pseudowork-energy principle is a useful tool for students solving problem exercises where the usual work-energy principle is very difficult to apply because of, e.g., complex rotational and/or vibrational motion involving both external and internal forces. Furthermore, as a pedagogical tool, the discussion of the pseudowork principle requires students to take a close second look at the work-energy principle, and leads to a better understanding thereof.

APPENDIX

As an example of the use of Eq. (5a), we consider the following problem: the automobile in Fig. 1 has a total mass $M = 1.5 \times 10^3$ kg, has four wheel drive, and tires whose coefficient of sliding friction on the road is $\mu_k = 0.9$. In a drag race, the driver accelerates the car so that all four wheels spin. What distance S must the car go to attain a final speed $v_{\text{cm}f} = 30$ m/sec? Solution

$$\text{Pseudowork} = \text{KE}_{\text{cm}}$$

$$\mu_k MgS = \tfrac{1}{2} M v_{\text{cm}f}^2$$

$$S = v_{\text{cm}f}^2/(2\mu_k g) = 51 \text{ m}.$$

This solution uses ideas of energy and "work" without reference to internal forces, rotational energy, or time. A more standard solution would make use of dynamics with Eq. (2), and essentially rederive our general starting point, Eq. (5a), for this special case.

[a] Supported in part by the NSF.

[1] F. W. Sears and M. W. Zemansky, *University Physics*, 4th ed. (Addison-Wesley, Reading, MA 1970).
[2] D. Halliday and R. Resnick, *Physics* (Parts I and II) (Wiley, New York, 1966).

Generalized equipartition

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In a recent note[1] it was pointed out that the "usual" equipartition theorem can be generalized. Here we wish to remind readers that this result is in fact a special case of a further generalization.

In fact, let z be one of the generalized coordinates or momenta $q_1, \ldots q_f, p_1, \ldots p_f$ of which the energy $E(q_1, \ldots, p_f)$ is a given function. Then if E contains a term $\epsilon \equiv az^r$, then

$$\langle \epsilon \rangle = kT/r. \qquad (1)$$

The "usual" result occurs for $r = 2$.

If $z \exp -E(q_1, \ldots, p_f)/kT$ vanishes at the limits of the integrations required in performing an average, then it can be shown[2] that

$$\langle z\partial E/\partial z \rangle = kT. \qquad (2)$$

The decomposition of E into $\epsilon = az^r$ plus terms independent of z, leads from Eq. (2) back to Eq. (1). The generalization (2) is of particular use in special relativistic problems.

[1] L. E. Turner, Jr., Am. J. Phys. **44** 104 (1976).
[2] P. T. Landsberg, *Thermodynamics with Quantum Statistical Illustrations* (Wiley, New York, 1961), pp. 288, 422.