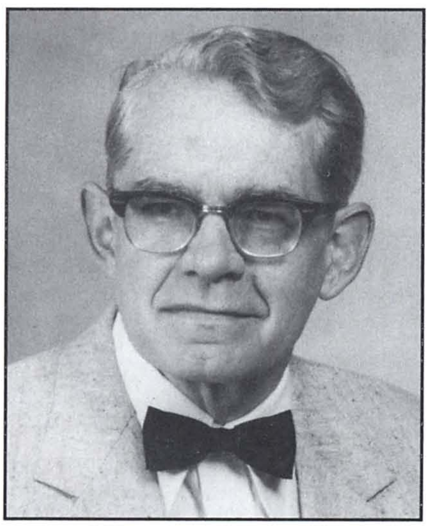
Physics that Textbook Writers Usually Get Wrong

I. Work

By Robert P. Bauman



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Physicists pride themselves on their precision of language, without which scientific communication breaks down, and on the consistency of physics. Is it possible to teach this kind of language and physics to students?

As a matter of expediency, we accept textbooks that contain major inconsistencies, often in the guise of simplification but also often for lack of adequate care in defining terms. In this article we examine problems that commonly appear in the definition and discussion of *work*.

We start with a restatement of two well-known ideas about work. First, work is a mode of transferring energy. Specifically, work is a transfer of directed energy (as contrasted with random energy, as in thermal conduction) to a defined system from the surroundings. Hence, if this is the only energy transfer,[1]

$$\Delta E = W \tag{1}$$

Second, work is accomplished by a force acting through a distance. More specifically,

$$W = \int f_i \cdot dx_i \tag{2}$$

or, for a constant force,

$$W = f_i \cdot \Delta x_i \tag{3}$$

That is, work is the integral of the scalar product of the force, f_i exerted on the system by an agent in the surroundings, and the displacement of the point of application of that force.

We will see that each of these statements is necessary to understand the definition of work. If either is neglected, errors are introduced.

Consider, now, some of the ways physics students are actually told about work.

Work-Energy Theorem

The most common statement of the work-energy theorem is:

The work done on an object is equal to the change in its kinetic energy.

Why does this introduce errors? Consider four examples.

1. An ideal gas is compressed isothermally.

$$W = -\int P\,dV = -nRT\ln\frac{V_2}{V_1} > 0$$

$$\Delta(K.E.) = 0$$

$$W \neq \Delta(K.E.) \qquad (4)$$

The gas as a whole does not move, so the kinetic energy is zero before and after the compression. (Also the average kinetic energy of the individual molecules does not change at constant temperature, although this is irrelevant to the work-energy theorem.)

Note in particular that there is no acceleration of the gas, so the net force on the gas is zero. If we were to mistakenly sum the forces *before* multiplying each by its displacement, we would get an incorrect answer.

$$W = \sum_i [\int f_i \cdot dx_i] > 0$$

$$W \neq \int [\sum_i f_i] \cdot dx = 0 \qquad (5)$$

2. A bicycle wheel is rotated in place, from rest to a final angular speed of ω. If the wheel has moment of inertia I, then

$$W = \Delta(\frac{1}{2} I\omega^2) > 0$$

$$\Delta(K.E.) = \Delta(\frac{1}{2} m v^2) = 0$$

$$W \neq \Delta(K.E.) \qquad (6)$$

3. A wad of gum is thrown against the wall.

$$W = 0$$

$$\Delta(K.E.) = -\frac{1}{2} m v_o^2 < 0$$

$$W \neq \Delta(K.E.) \qquad (7)$$

No energy is transferred to the wall in the collision because the point of application of the force, exerted by the wall on the gum, does not move.[2] The kinetic energy of the gum goes into internal energy of the gum.

This example can be demonstrated effectively with "happy" and "unhappy" (or bounce and no-bounce) balls. The ball that does not bounce converts kinetic energy into internal energy.

4. A student jumps for joy over success in physics.

$$W = 0$$

$$\Delta(K.E.) = \frac{1}{2} m v_f^2 > 0$$

$$W \neq \Delta(K.E.) \qquad (8)$$

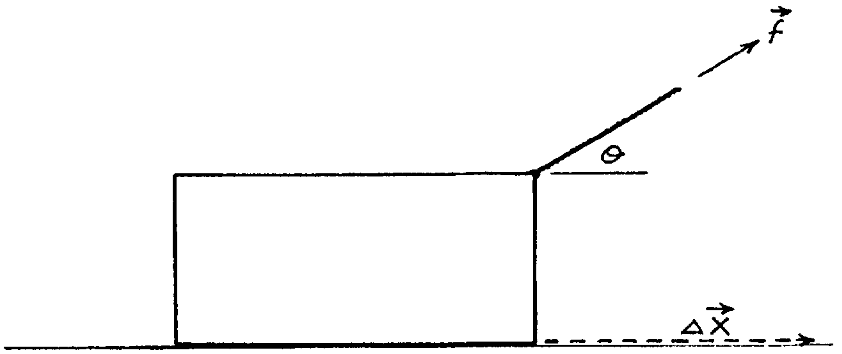

Fig. 1. Work done by the person on the cord and work done by the cord on the block are easily calculated: $W = f\Delta x\cos\theta$. Work done by the block on the floor is not operationally defined.

The student cannot extract energy from the floor. The energy to jump came from food previously eaten. The point of application of the force, exerted by the floor, did not move.

The true work-energy theorem is quite unremarkable. It simply says that if energy is transferred to an object (as work) *and* if there is no place for the energy to go except into kinetic energy, then

$$W = \Delta(K.E.) \qquad (9)$$

This is more simply (but apparently obscurely) stated as:

The work done on a particle is equal to the change in kinetic energy of the particle.

A particle is defined as an object, of any size, that does not change its internal properties or rotational state. Hence the Earth or Moon are often treated as particles, when tidal forces can be neglected.

Nonoperationally Defined Work: Friction and Radiation

A frequent statement in textbooks, made to emphasize the first part of the definition of work [Eq. (1)] is:

If an object does not move, no work is done.

This is often followed by a valid example, such as a student pushing against a wall. Unfortunately, the statement violates the second part of the definition of work [Eq. (2)].

1. Consider the familiar problem of a block pulled a distance Δx across the floor by a force, f, applied to a cord (Fig. 1). The person pulling the cord does work on the cord, equal to the component of the force in the direction of motion times the distance the end of the cord moves.

$$W_{p/c} = f\Delta x\cos\theta > 0 \qquad (10)$$

The cord cannot store this energy. It passes it along to the block, doing work equal to the component of the force in the direction of motion times the distance the end of the cord moves.

$$W_{c/b} = f\Delta x\cos\theta > 0 \qquad (11)$$

The net work done on the cord is zero.

The block cannot store all of this energy. The block moving at constant speed (unaccelerated) passes energy along to the floor. The block is moving and there is a frictional force exerted by the floor on the block, so the work done by the floor on the moving block is usually calculated as

$$f_{friction} \cdot \Delta x = -f_{friction} \Delta x < 0 \qquad (12)$$

because the force of friction is opposite in direction to the displacement (but see below).

If the work done by the floor on the moving block is negative, then the work done by the moving block on the stationary floor is positive. Work is done on the floor. Energy is transferred from the block to the floor, even though the floor is "not moving."

2. The work done by frictional forces can be explored more effectively by considering another familiar problem. A block slides down an inclined plane at constant speed (Fig. 2). If the block is not accelerated, the net force on the block is obviously zero, from Newton's second law.

Treating the block as a particle, the net work done on the block, $\int f\,dx$, must be zero. The speed, and therefore the kinetic energy of the block, is the same at the bottom of the incline as at the top.

This problem requires examination in more detail. Energy is being transferred to the block from the gravitational field.

$$W_{g/b} = \Delta E = m\,g \cdot \Delta h = -m\,g\,\Delta h > 0 \qquad (13)$$

If the block were falling freely, it would gain this energy as kinetic energy:

$$\Delta(K.E.) = -m\,g\,\Delta h > 0 \qquad (14)$$

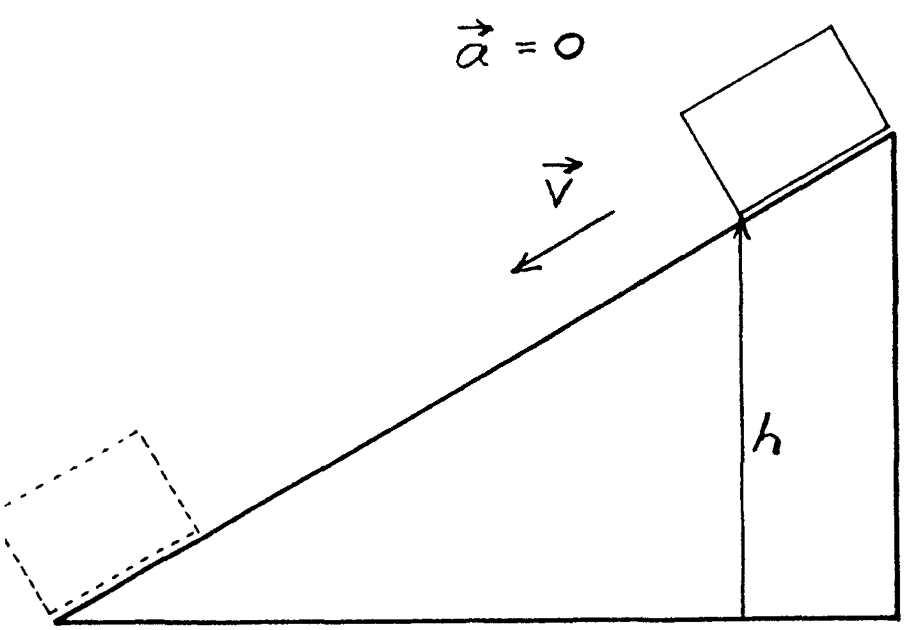

Fig. 2. The block moves down the slope at constant speed, impeded by friction with the surface. Work done by the surface on the block is negative, but not operationally defined.

But the block is exerting a force on the slope, so the block is transferring energy, as work, to the slope, leaving the block, considered as a particle, with no net change in energy. If the process is reasonably rapid, little or no thermal energy is transferred ($Q \approx 0$), because transfer of thermal energy is a slow process.

As for the block being pulled across the floor, the frictional force is opposite in direction to the displacement, so a negative amount of work is done by the slope on the moving block (i.e., energy is transferred from the block to the slope), and therefore a positive amount of work is done by the moving block on the stationary slope.[3]

However, the initial analysis was too simple, because although the gravitational field transferred energy to the block and the block transferred energy to the slope, the block is warmer at the bottom than at the top. The amount of energy transferred by the block to the slope is *not* equal to $\int f\,dx$, where f is the net force exerted by one on the other and dx is the change in position of the block.

Sherwood and Bernard[4] have shown that the important idea is that the point of application of the frictional force does move, but it moves in unpredictable fashion. A simple analogy illustrates the principle. Consider a moving block coming in for an aircraft-carrier type landing (Fig. 3). It is snagged by a spring, and transfers its kinetic energy to the spring, which stores the energy temporarily as potential energy. The spring could return this energy to the block, sending it back where it came from. But if the spring is detached from the block while stretched, and the spring is regarded as part of the surroundings, the energy of the block has been totally transferred to the surroundings.

On the other hand, if the spring is regarded as part of the block, and is detached from the surface while stretched, the energy of the block remains with the block, but is now converted to thermal energy within the system as the spring oscillates and comes to rest.

A more general point of view, consistent with what we know about friction, would be that the spring might be broken in the middle, transferring half of the energy to the surroundings, or a collection of springs might break at various points, giving any possible answer to the question of how much energy is transferred from the block to the surface. Work cannot be calculated when friction forces are acting. Unless Q is known (which is unlikely), W cannot be measured. It is not operationally defined.

3. Radiation emitted by a hot object is not transferred as work, but rather as "heat" or, better, thermal energy transfer, Q. Similarly, sunlight striking a car warms the car, without any evidence of doing work on the car. However, if photons from the Sun strike a properly pre-

pared silicon surface, an electric current is produced that can lift a weight, charge a battery, or do other forms of work.

When is radiation transfer Q and when is it W? The answer emphasizes, again, the need for an operational definition, which is generally lacking. In broad terms, however, it seems clear that transfer of radiation between any object and a radiation field in equilibrium with the object (that is, black-body radiation or absorption) is entirely Q. Radiation transfer from a radiation field to or from an object *not* in equilibrium with the field (such as solar radiation on a photovoltaic surface or laser emission) may be a mixture of Q and W.[5]

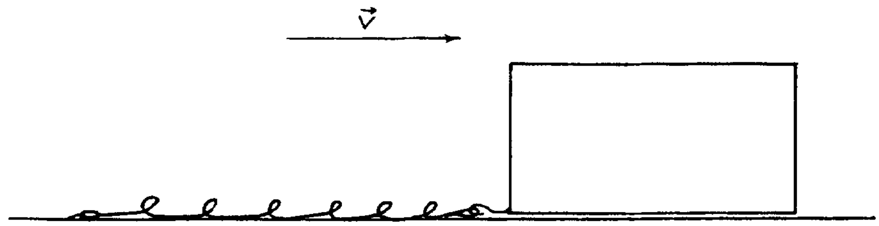

Fig. 3. Kinetic energy of the moving block is converted to potential energy in the spring, then degraded to thermal energy. Depending on where the spring separates, all, part, or none of the energy is transferred as work to the surroundings.

Potential Energy

When energy is transferred to a system, as work, and stored in a form such that the energy may be recovered as work (in an isothermal process), the energy is said to be stored as "potential energy." For example, when a spring of force constant k is stretched or compressed through a distance x, work is done on the spring, increasing its energy by $1/2\, k x^2$, so the spring is said to possess potential energy.

$$W = \Delta(P.E.) = \frac{1}{2} k x^2 \tag{15}$$

1. Now consider the following legerdemain. Assume

$$K.E. + P.E. = constant \tag{16}$$

$$W = \Delta(K.E.) \tag{17}$$

then

$$W = -\Delta(P.E.) \tag{18}$$

Equation (18) tells the student that energy transferred to the system, as work, *decreases* the energy of the system. Note, however, that Eq. (16) assumes no energy transfer to or from the system, so $W = 0$. Equation (17) is the misapplied work-energy theorem, which does not fit here because energy can be stored as potential energy as well as kinetic energy. Consequently, Eq. (18) has a minus sign where there should be a plus sign [to be consistent in sign convention with Eq. (17)]. The errors are noncompensating, so the answer is off by twice the value.

Unfortunately, the preceding sequence of equations is not a creation of this author's fertile imagination during a sleepless night. It is found in some of the best-selling freshman physics textbooks.

Before we cast stones at textbook authors, however, consider the kind of physics we expect from our textbooks (and which the authors therefore feel obligated to deliver).

2. A ball is thrown upward with initial speed v_o. If m = 0.50 kg and v_o = 10 m/s, the initial kinetic energy of the ball is $1/2\, m\, v_o^2$ = 25 J. This energy is converted to potential energy, so at the top of its motion (approximately 5 m) the ball has potential energy of 25 J and zero kinetic energy. As the ball falls, the potential energy is converted to kinetic energy, giving zero final potential energy and final kinetic energy = 25 J. This is the standard textbook analysis.

Now examine the problem more carefully by modifying it slightly. Raise the ball (considered as the system), with your hand, at constant speed (which may be arbitrarily slow to avoid concern with accelerations, even though these would compensate at bottom and top). Lift the ball to approximately 5 m, as before. During this process, two forces are at work. Your hand is exerting an upward force on the ball, $f_{hb} = m\, g \approx 5$ N. The gravitational field is exerting a downward force on the ball, $f_{gb} = -m\, g$ (or $f = m\, g$, where g is directed downward). The corresponding work terms are

$$W_{hb} = m\, g\, h$$
$$W_{gb} = -m\, g\, h$$
$$W_{total} = 0 \tag{19}$$

The net force on the ball (which acted as a particle) was zero at all points in the motion, so no work was done on the ball. The kinetic energy of the ball did not change, the internal energy of the ball did not change, and the potential energy of the ball did not change.

What did happen? You transferred energy to the ball ($W > 0$) and the ball transferred energy to the field ($W < 0$). The potential energy is stored in the field.

Now we release the ball from approximately 5 m. The field exerts a force on the ball, $m\, g$, in the direction of the motion ($W = m\, g \cdot \Delta h$, where g and Δh are both downward), giving the ball kinetic energy at the bottom of the motion of 25 J.

$$W = m\, g\, h = 25\text{ J} \qquad \Delta(K.E.) = 25\text{ J} \tag{20}$$

If we attribute potential energy to the ball at the top, we cannot properly calculate the work done on, and hence energy transferred to, the ball in its descent. We open the door to "patch work" solutions such as Eq. (18).

It is, of course, convenient to think of objects in gravitational (or electric) fields as having potential energy. We can do that by redefining the system as ball plus field. Then the field can do no work on our system, as the ball goes up or down.

What must be done is a) to be consistent, within any problem, in how we define the system and hence how work is calculated, and b) let students understand which way we have chosen to define the system in any given problem.

Inconsistent Terminology

The beauty, and utility, of a consistent science is that it can be tested at any point. Any inconsistency tells us that there is an error somewhere in our structure, like a spider sensing an intruder or prey by vibrations in the web. Unlike the web, though, which can sustain damage and still be quite functional, an inconsistent science loses its simplicity, creating uncertainty where there should be none.

Part of the difficulty with work seems to be an embedded attachment of the term *work* to the concept of *force*. A powerful pedagogical message is: *Forces are free*.

Any reasonable magnitude and/or direction of force can be provided, for an arbitrary length of time, at no cost. For example, a downward force is supplied by a box or truck. An upward force is supplied by the pavement under the box or truck. With levers, pulleys, or wedges, a vertical force downward may be converted to a vertical force upward, or vice versa, or to a horizontal force.

It is only when the point of application of the force moves that energy is transferred; it is the energy transfer that represents an expenditure. By contrast, change in momentum is the *net* force acting on an object multiplied by *time*, so the floor acting on a bouncing ball can change the momentum of the ball ($\Delta p = \int f\,dt = -2\,mv_o$), even though no work is done on the ball ($W = \int f\,dx = 0$).

What, then, do we do with terms that do not fit? For example, what is "internal work"? It cannot be energy transferred to the system, from the surroundings, as work. It cannot be calculated as the scalar product of the force applied to the system by an external agent times the displacement of the point of application of that force. In short, it has nothing to do with work, and should not carry a title that implies that it is a form of work.

More recently, in useful analyses of operational definitions of work,[6] it has been pointed out that the momentum-kinetic energy equation, involving the net force and the displacement of the center of mass, $\int f_{net}\,dx_{cm}$, superficially resembles an expression for work (but then so also does the expression for torque, $\tau = f\ell$ where f and ℓ are mutually perpendicular).

Let f_n be the net force acting on the system and x_{cm} and v_{cm} describe the position and speed of the center of mass of the system, so that $v_{cm} = dx_{cm}/dt$ and $f_n = m\,a_{cm} = m\,dv_{cm}/dt$. Then

$$\int f_n\,dx_{cm} = \int f_n \frac{dx_{cm}}{dt}\,dt =$$

$$\int m\frac{dv_{cm}}{dt}v_{cm}\,dt = \int m\,v_{cm}\,dv_{cm} = \Delta(\frac{1}{2}m\,v_{cm}^{\ 2}) \qquad (21)$$

The mathematics of Eq. (21) are correct, but the first integral is *not*, in general, equal to the work done. Why, then, is the answer right?

We should recognize that *net force* and motion of *center of mass* are typical of calculations involving momentum. The equation as given is always correct, because it is an integral of the *momentum* of the system. More explicitly, the integral is proportional to the change in the square of the momentum.

$$\int f_{net}\cdot dx_{cm} = \int m\,v_{cm}\,dv_{cm} =$$

$$\int \frac{1}{m}p\,dp = \frac{1}{m}\Delta(\frac{p^2}{2}) \qquad (22)$$

Only if all forces acting on the system act on points undergoing the same displacement as the center of mass of the system will this integral be equal to work done on the system. That is, the calculated change in kinetic energy is equal to the work done on the object only if the object acts as a particle. Equation (21) or (22) may be labeled properly "the momentum-kinetic energy equation," but it has no intrinsic connection with work. Use of a name for this integral that contains the word "work" has already caused confusion and will continue to do so for coming generations of students if the terminology persists.

Summary

The concept of work is easy if it is presented in a self-consistent way. The definition of work includes two critical parts: work is a transfer of energy between system and surroundings, and work is an integral of force times the displacement (in the direction of the force) of the point of application of that force. It is necessary to decide what is the system under consideration and whether energy is being transferred to or from that system.

The so-called work-energy theorem is overemphasized. It says that when energy is transferred to a system and the system can gain only kinetic energy, then the energy added appears as kinetic energy. When the terminology is applied for systems that can gain other forms of energy, errors are introduced.

Work is a convenient idealization. It is operationally defined only for idealized interactions, not involving friction or absorption of radiation under nonequilibrium conditions. That does not make it any less useful for analyzing the friction-free problems, or problem parts, of introductory physics courses.

It is critically important to define a system whenever the concept of work is invoked. Lack of definition of a system is particularly dangerous when considering potential energy, for which sign errors and other inconsistencies often appear.

Not everything that looks like work is work. If attention is paid to the two parts of the definition, such terms as "internal work," and therefore also "external work" (as if there were something else), will disappear. The momentum-kinetic energy equation is particularly likely to entrap the unwary because it bears a strong resemblance to work but is usually not equal to work.

When we are careless with definitions or try to tamper with the truth, making physics "a little bit wrong" in the name of simplification, we destroy the self-consistency of physics and make it so difficult that only memorization can provide the answers expected by the instructor.

Notes and References

1. Sometimes work is defined as energy transferred from the system to the surroundings, so $\Delta E = -W$. This choice of convention changes the language, and intermediate signs, but does not change the conclusions. Especially in introductory physics, the sign convention chosen here is much more common.
2. Even if there is some deflection of the wall, it is small and the force is finite, $\vec{f_i} \cdot \Delta \vec{x_i} < \Delta (K.E.)$.
3. It is tempting to leap to the conclusion that it is only relative motion that determines work. However, it is well known that energy depends on the frame of reference, even for unaccelerated frames. The transfer of energy also depends on the reference frame of the observer. The critical issue is the motion, in the reference frame of the observer, of the point of application of the force.
4. B.A. Sherwood and W.H. Bernard, "Work and heat transfer in the presence of sliding friction," *Am. J. Phys.* **52**, 1001 (1984).
5. R.P. Bauman, *Modern Thermodynamics with Statistical Mechanics* (Macmillan, New York, 1992), p. 182.
6. The point is discussed by C.M. Penchina, "Pseudowork-energy principle," *Am. J. Phys.* **46**, 295 (1978) and by Sherwood and Bernard, op. cit., but the description as "pseudowork" obscures the simplicity of the argument.

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