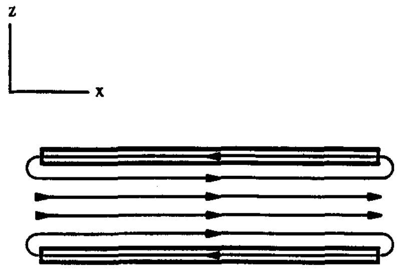

Fig. 2. Half of the energy which is moving within the field from left to right flows back to the left through the plates of the capacitor.

section exerts on the part right of it. One might ask, however, where this force originates: How is the field able to exert a force on the left side of the plates, and how can the right part of the plates exert a force on the field? The answer can only be: By means of the stray field. Indeed, only the stray field at both ends of the plates has components parallel to the plate direction and is therefore able to pull outwards.[11]

Inserting (7) in (6) we get the energy flow within the plates:

$$P'' = -(\epsilon_0/2)|\mathbf{E}|^2|\mathbf{v}_\parallel|ld. \qquad (8)$$

Now the energy balance is restored. Indeed, by using Eqs. (4), (5), and (8) it can be seen that

$$\frac{dW}{dt} = P' + P''.$$

Half of the total energy current which goes from left to right in the field flows back through the plates and the other half serves to transfer the electrostatic field from left to right, see Fig. 2. That part of the energy current within the field which goes back through the plates has to make 180° turns within the stray field on the right side of the capacitor in order to get into the plates, and it must make additional turns when it leaves the plates on the left side of the capacitor. Here again, one recognizes the importance of the often neglected stray field.

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[11]If forces are interpreted as momentum currents and mechanical stress as momentum current density, then it can be seen that we simply have closed loops of momentum currents: Momentum is flowing from left to right within the field. It then makes 180° turns within the stray field and enters the plates. It then flows back to the left within the plates. At the left end it leaves the plates, enters the field, makes again 180° turns within the stray field region and flows to the right within the homogeneous part of the field. See F. Herrmann and G. Bruno Schmid, "Momentum flow in the electromagnetic field," Am. J. Phys. **53**, 415–420 (1985).

Stopping objects with zero external work: Mechanics meets thermodynamics

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Although the work-energy theorem of *pure, nondissipative* mechanics states that the work done stopping a body equals its kinetic energy change, the work done stopping a body via an *inelastic, dissipative* collision can be zero. This counter-intuitive result is used to motivate the development of thermodynamic ideas as a direct extension of classical mechanics. The approach leads to a natural introduction of internal energy, the path dependence of work, and dissipation. It also offers an opportunity for early exposure to powerful symmetry and frame-invariance arguments. The main presentation addresses one-dimensional highly symmetric collisions, with a generalization in the Appendix.

I. INTRODUCTION

Students of classical mechanics learn from the work-energy theorem that negative external work is required to stop a moving object. In contrast, our intentionally provocative title suggests the possibility of stopping an object while doing *zero* external work. Informal inquiries of col-

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leagues have shown us that many physics teachers find this notion hard to believe. The second part of our title indicates *why* the zero-work idea is hard to accept: *it occurs in a realm where both mechanics and thermodynamics are involved.* Evidently, physics teachers (and probably their students) commonly make the default assumption that object stopping is a purely mechanical process.

That this is *not* the case can be seen by considering an idealized totally inelastic collision between a ball of putty and a perfectly smooth, flat wall in a zero gravity environment. Because the wall is rigid, the forces on the ball perpendicular to the wall do zero work. And because there is no friction, zero work is associated with any plastic displacements of the ball parallel to the wall's plane. Therefore, the ball is stopped with zero *external* work. As the ball slows down, parts of it continue moving toward the wall and *internal* work converts bulk kinetic energy to *internal* energy, resulting in a temperature increase.

A significant body of literature on energy transformations in macroscopic systems already exists. Recently, Arons[1] gave an articulate and convincing exposition on the need for thermodynamic ideas to correctly describe the mechanics of macroscopic systems. He reviewed and elaborated on articles[2–5] dealing with internal work, the pseudowork-energy theorem (pseudowork is also called center-of-mass work[6]), and sliding friction phenomena. Arons wrote: "Since these papers...have not received much attention and seem to have had little influence on the textbooks, I present this review...in the hope that it will generate somewhat wider interest and lead to a conceptually sounder treatment of the energy concepts for beginning students." In an attempt to clarify these matters further, we developed a taxonomy of work,[7] identifying seven types of work that can be done on a system of particles, showing connections between them, and relating them to specific energy changes. Other worthwhile contributions ranging from the very definition of energy[8,9] to the distinctions between work, heat, and internal energy exist in the physics-teaching literature.[10–22] These efforts have influenced our thinking and suggest that avoidance of thermodynamics in discussions of mechanics misses an opportunity to address familiar, fascinating, and essential aspects of physics and gives students an incomplete and inadequate view of our physical world.

In this paper, we motivate the development of thermodynamic ideas as an extension of classical mechanics. Our emphasis is on a one-dimensional head-on, totally inelastic, mirror-symmetric (in the center-of-mass frame) collision between two bodies. This model offers a previously unappreciated and, to our knowledge, unexplored opportunity for an early introduction to thermodynamics concepts in an attention-grabbing context. It enables early student exposure to the important concepts of internal energy, frame invariance, path dependence of work, symmetry arguments, and dissipation.

The remainder of this paper is organized as follows. We show in Sec. II that pure mechanics cannot correctly describe the one-dimensional inelastic collision model and prove, using a symmetry argument, that the external work on each of the colliding bodies must be zero. In Sec. III we then argue that a new term must be added to the work-energy theorem in order to: (1) account for the internal structure of macroscopic objects, and (2) explain the zero-work result. We show in Sec. IV that this added energy

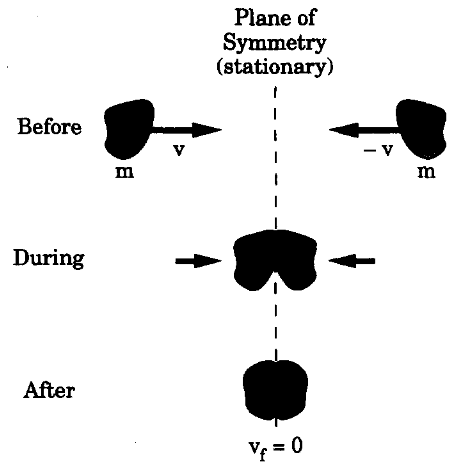

Fig. 1. Head-on, totally inelastic collision between two mirror-symmetric, equal-mass bodies in the center-of-mass reference frame. Initially the bodies have equal and opposite velocities v and $-v$. During the collision both bodies deform and slow down symmetrically. Note, in particular, that *during* the collision the interface between the two bodies lies, at all times, in the (CM) plane of symmetry. After the collision the bodies form one object of mass 2m with zero velocity.

change has the same value in *any* inertial frame—an expected characteristic of an *internal* property—and can be viewed as an *internal* energy change. We summarize and interpret our main findings and their generalizations in Sec. V and extend our analysis to general asymmetric, multidimensional two-body collisions with arbitrary inelasticity in the Appendix. Readers who wish to scan the major findings are directed to results 1–5 and the associated discussion in that section.

II. A SIMPLE EXAMPLE WITH A COUNTER-INTUITIVE RESULT

Consider a mirror-symmetric, totally inelastic collision between two bodies in the system's center-of-mass frame (CM frame) as shown in Fig. 1. We assume the initial direction of each body is horizontal and define mirror symmetry as left–right symmetry about a vertical plane through the center of mass. This symmetry implies the bodies have the same mass, m, equal and opposite initial velocities ($+v$ and $-v$), and mirror-inverted internal structures with respect to the vertical plane. Because the collision is totally inelastic and the total linear momentum is zero, each body comes to rest during the collision. The translational kinetic energy changes of the bodies are

$$\Delta K_1 \equiv K_{1f} - K_{1i} = -(1/2)mv^2 \tag{1a}$$

and

$$\Delta K_2 \equiv K_{2f} - K_{2i} = -(1/2)mv^2, \tag{1b}$$

where the superscripts i and f stand for "initial" and "final" and 1 and 2 refer to bodies 1 and 2.

The total change in translational kinetic energy for the two-body system is

$$\Delta K_{\text{sys}} \equiv \Delta K_1 + \Delta K_2 = -mv^2. \tag{2}$$

Where does the initial kinetic energy go? The answer from most teachers is likely to be something like, "to heat" or "to thermal energy." Although not strictly wrong, these replies mask a good deal of interesting physics, which we now pursue.

The standard work-energy theorem of mechanics[6] relates the total external work W_{ext} done on a body to its translational kinetic energy change:

$$W_{ext}=\Delta K \quad \textit{Work-energy theorem of mechanics.} \tag{3}$$

Applying Eq. (3) to the bodies in this collision, where only translational motion occurs, we find

$$W_{ext,1}=\Delta K_1=-(1/2)mv^2 \tag{4a}$$

and

$$W_{ext,2}=\Delta K_2=-(1/2)mv^2. \tag{4b}$$

Here, $W_{ext,1}$ and $W_{ext,2}$ are the works done on bodies 1 and 2, respectively, by bodies 2 and 1. The results of Eqs. (4) are consistent with the assumed symmetry of the collision, which dictates that

$$W_{ext,1}=W_{ext,2} \quad \textit{By symmetry.} \tag{5}$$

We now make the additional assumption that the two bodies interact exclusively via forces that act only at surfaces in physical contact. The interface between the two bodies forms a macroscopic "contact surface" which, in general, can vary in size and shape during a collision. Two facts are implied by the assumed symmetry (see Fig. 1). First, the contact surface is confined at all times to the plane of symmetry because any penetration through that plane by either body would violate the assumed symmetry. Second, the interaction forces must be *perpendicular* to the plane of symmetry or Newton's third law would imply an action-reaction force pair that violates the assumed symmetry. Therefore,

$$W_{ext,1}=W_{ext,2}=0 \quad \textit{Zero-work result.} \tag{6}$$

Equation (6) shows that *each object does zero work while stopping the other.* This zero-work result reflects the fact that the interaction forces are always perpendicular to the displacements (if any) of the elements on which they act. Unfortunately Eqs. (4) are incompatible with Eq. (6). We shall see presently that the difficulty is with the work-energy theorem of mechanics, Eq. (3).

The zero work result is physically "obvious" for the special case where the two bodies move together (have equal displacements) at each point on the contact surface; i.e., when there is zero slippage. Then Newton's third law implies that

$$W_{ext,1}=-W_{ext,2} \quad \textit{Assuming zero-slip contact forces.} \tag{7}$$

The zero work result, Eq. (6), then follows from the combination of the symmetry result, Eq. (5), and the assumption of zero-slip contact forces.

Given the zero external work, what is the *physical mechanism* by which stopping occurs? It is simply that the net external force $F_{ext,1}(t)$ on body 1 generates a corresponding impulse,

$$I_1=\int_{\text{collision}} F_{ext,1}(t)dt=-mv, \tag{8}$$

where $-mv$ is the momentum change of body 1. Viewed in terms of momentum rather than energy, it is clear that $F_{ext,1}(t)$ is responsible for stopping body 1 even though it does zero work in the process. Similarly, the impulse on body 2 is $I_2=+mv$.

III. LOOKING INWARD

A bit of reflection shows why Eqs. (4) and (6) are incompatible. The work-energy theorem holds in *pure* mechanics and applies only to point particles or (impossibly) ideal rigid bodies. It requires modification in problems where bulk mechanical (kinetic or potential) energy is transformed into nonmechanical *internal* energy associated with the molecules of macroscopic bodies. This nonmechanical energy is present even in objects at rest. It is an *internal* energy that is "invisible" from a purely mechanical point of view—a characteristic that makes it somewhat mysterious, but also quite interesting. If a collision excites internal degrees of freedom, increasing internal energies, the work energy theorem must be modified.

We may remove the interpretive problems and inconsistency encountered above by *postulating* that Eq. (3) be replaced by

$$W_{ext}=\Delta K+\Delta U \quad \textit{Extended work-energy theorem,} \tag{9}$$

in which W_{ext} is the *net external work* done on the body and ΔU is the *internal* energy change of the body. The extended theorem accounts for the fact that energy transferred to a body can shown up as translational kinetic energy and/or in a variety of internal storage modes. Generally, U includes energies due to bulk rotation and vibration as well as "hidden" modes that are within the domain of thermodynamics. Furthermore, Eq. (9) allows processes for which bulk kinetic energy is transformed to or from these internal modes, whether or not any external work is done. Applying Eq. (9) to the individual bodies in our mirror-symmetric collision, we find, using (1) and (6),

$$W_{ext,1}=\Delta K_1+\Delta U_1=0 \tag{10a}$$

and

$$W_{ext,2}=\Delta K_2+\Delta U_2=0, \tag{10b}$$

or

$$\Delta U_1=\Delta U_2=(1/2)mv^2. \tag{11}$$

This conforms with the assumed symmetry and has the pleasing interpretation that the original kinetic energy of the two bodies is transformed completely into internal energy during the collision.

Equation (9) tells us that the work done on body 1 consists of two cancelling terms: $\Delta K_1=-(1/2)mv^2$ is formally equal to the (negative) *pseudowork* or *center-of-mass work*;[3,4,6] and $\Delta U_1=(1/2)mv^2$ is a (positive) change in internal energy attributable to deformation during the collision. These two terms have equal magnitudes and opposite algebraic signs. Given this interpretation, it is important to avoid a tempting mistake. Neither ΔK_1 nor ΔU_1 is attributable to work done by body 2 because that work is *identically zero*! Both terms *can* be attributed to the force $F_{ext,1}$ on body 1. Prior to the collision, when $F_{ext,1}=0$,

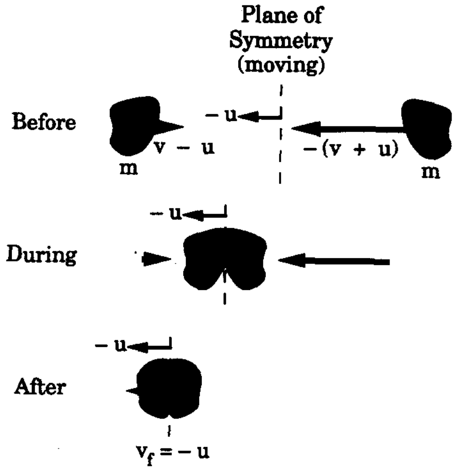

Fig. 2. The collision of Fig. 1 depicted in a "u frame" moving with velocity u (>0) with respect to the center-of-mass reference frame. Before the collision the bodies have velocities $v-u$ and $-(v+u)$. During the collision the bodies deform and the interface plane moves with constant velocity $-u$. After the collision the bodies form one object of mass 2m with velocity $-u$.

there exists no mechanism for converting translational kinetic to internal energy. Once contact is made, the nonzero force $F_{\text{ext},1}$ *constrains* the motion of body 1, decreasing its momentum *and* coupling its bulk kinetic and internal energies. Specifically, work by *internal* forces transforms bulk translational kinetic energy into internal energy. *All* the work done stopping body 1 is done internally *by* body 1 molecules *on* body 1 molecules. A similar statement holds for body 2.

IV. IS THE INTERNAL ENERGY CHANGE REALLY "INTERNAL"?

The concepts of internal energy and internal work are somewhat esoteric and we can benefit from a deeper understanding of their nature. So far, our analysis has been confined to the CM frame. The works performed on the bodies and their translational kinetic energy changes are different in a frame moving relative to the CM frame. But what about the changes in the bodies' internal energies? We answer this by looking at the same collision in a "u-frame," namely, a frame moving with velocity u (parallel to the line of motion of the bodies) with respect to the CM frame, as shown in Fig. 2.

Before the collision, the velocities of bodies 1 and 2 in the u frame are $(v-u)$ and $-(v+u)$, respectively. After the collision both bodies move with velocity, $-u$. The corresponding initial and final kinetic energies are

$$K_{1i}(u)=(1/2)m(v-u)^2,\quad K_{2i}(u)=(1/2)m(v+u)^2 \tag{12a}$$

and

$$K_{1f}(u)=K_{2f}(u)=(1/2)mu^2, \tag{12b}$$

showing that the individual kinetic energies are frame dependent. The kinetic energy changes are

$$\Delta K_1(u)=-(1/2)mv^2+mvu \tag{13a}$$

and

$$\Delta K_2(u)=-(1/2)mv^2-mvu. \tag{13b}$$

Thus the individual kinetic energy *changes* are also frame dependent. Adding the individual kinetic energy *changes* we find for the two-body system:

$$\Delta K_{\text{sys}}(u)\equiv\Delta K_1(u)+\Delta K_2(u)=-mv^2, \tag{14}$$

showing that the total amount of kinetic energy that "disappears" is the same in *any* frame. This shows that *some* collision properties are *internal* characteristics of the system that are not influenced by the (irrelevant) motion of the observer.

We now examine how the work on each body varies with our choice of reference frame. In the u frame, the contact surface moves at constant velocity, $-u$. Body 1 experiences a time-dependent force, $F_{\text{ext},1}$, due to its contact with body 2. Because of the symmetry constraint, $F_{\text{ext},1}$ acts parallel or antiparallel to the direction of motion of the contact surface. Body 2 experiences a force, $F_{\text{ext},2}$, from its contact with body 1, which is at all times equal and opposite to $F_{\text{ext},1}$ by Newton's third law. Accordingly, the works done on each body by the other are

$$W_{\text{ext},1}(u)=\int_{\text{collision}}F_{\text{ext},1}(t)(-u\,dt)$$
$$=-u\int_{\text{collision}}F_{\text{ext},1}(t)dt\equiv-uI_1 \tag{15a}$$

and

$$W_{\text{ext},2}(u)=\int_{\text{collision}}F_{\text{ext},2}(t)(-u\,dt)$$
$$=u\int_{\text{collision}}F_{\text{ext},1}(t)dt\equiv uI_1=-W_{\text{ext},1}(u), \tag{15b}$$

where, as before, I_1 is the impulse on body 1. Equation (15b) shows that Eq. (7) holds in any frame, as expected, because the arguments leading to Eq. (7) are themselves frame independent. According to Eq. (8), I_1 equals the momentum change of body 1; i.e., $m(v_{1f}-v_{1i})=-mv$ in *any* frame. Therefore Eqs. (15) reduce to

$$W_{\text{ext},1}(u)=mvu \tag{16a}$$

and

$$W_{\text{ext},2}(u)=-mvu. \tag{16b}$$

Finally, we apply Eqs. (9), (13), and (16) to each body:

$$W_{\text{ext},1}(u)=\Delta K_1(u)+\Delta U_1(u)\Rightarrow\Delta U_1(u)=(1/2)mv^2, \tag{17a}$$

and

$$W_{\text{ext},2}(u)=\Delta K_2(u)+\Delta U_2(u)\Rightarrow\Delta U_2(u)=(1/2)mv^2. \tag{17b}$$

Equations (17) show that *the internal energy changes are frame independent* because the frame transformations for work and kinetic energy change both entail terms of the form $\pm mvu$. The result is physically pleasing because it means that the internal energy changes are *really* "internal," as expected if each body stores an amount of internal energy that is independent of our reference frame.

V. RECAPITULATION AND DISCUSSION

Mirror-symmetric, one-dimensional, completely inelastic collisions can serve as a natural bridge between classical mechanics and thermodynamics. The failure of the simple work-energy theorem makes the internal energy concept a compelling necessity, and Galilean transformations to frames other than the CM frame illustrate that the internal energy change is a frame-independent quantity. Our major findings for this simple collision, together with generalizations obtained in the Appendix can be encapsulated in five major results.

Result 1. In a mirror-symmetric, completely inelastic, one-dimensional collision [see Fig. (1)], for which interactions are solely through contact forces, each body stops the other while doing *zero* work on it. The force on each body reduces its momentum but does zero work because each force is perpendicular to any displacements that occur. *Internal* work, via *internal* forces, converts each object's bulk translational energy into *internal* energy.

Result 2. For the mirror-symmetric collision, the work done on each body depends upon the observer's reference frame. The mirror symmetry in the CM frame is broken in frames with $u \neq 0$ and by varying u, the observed work done by either body on the other can be made negative or positive, with the zero value occurring only in the CM frame. Observers in different reference frames see different paths of motion and frame dependence corresponds to *path dependence*, a well-known characteristic of thermodynamic work.

Result 3. For the mirror-symmetric collision, three characteristic energy changes are independent of the observer's reference frame: the internal energy changes of each colliding body and the translational kinetic energy change of the two-body system. Frame invariance reflects the inherently *internal* nature of internal energy and facilitates an effective introduction of this thermodynamic concept as an outgrowth of classical mechanics.

Result 4. In the Appendix, we show that analogs of results 1–3 hold for general collisions between two *arbitrary* bodies interacting via contact forces, but not confined to one dimension, and with arbitrary inelasticity. Generally, the works on bodies 1 and 2 satisfy the condition $W_{\text{ext},1} = -W_{\text{ext},2}$ in *any* inertial frame and, if the collision alters the momenta of the two bodies, the two works vanish in a *family* of reference frames. The three frame-invariant energy changes in result 3 are invariant also in the general case.

Result 5. For inelastic collisions, *dissipation* entails a transformation of bulk translational energy into the internal energies of the bodies; i.e., *positive internal energy changes occur for both bodies.* This generally leads to temperature increases of the bodies that can induce heat transfers. It is helpful to envision an "adiabatic" collision, for which internal energies increase with zero heat transfer, followed by *secondary* heat transfers to the environment. Although oversimplified, this view helps sort out the important difference between internal energy increase and heat transfer, which are commonly confused with one another.

We now discuss the major results further. For our mirror-symmetric collision, the *physical origin* of the internal energy changes seems very different in different frames. For example, if $u = (1/2)v$, Eqs. (13) and (16) imply $\Delta K_1(u) = 0$ and $W_{\text{ext},1}(u) = \Delta U_1 = (1/2)mv^2$, suggesting that ΔU_1 is generated via *external* work by body 2 on body 1. In contrast, if $u = -(1/2)v$, then $\Delta K_2(u) = 0$, $\Delta K_1(u) = -mv^2$, and $W_{\text{ext},1}(u) = -(1/2)mv^2$, suggesting that the positive work needed to generate $\Delta U_1 > 0$ is *not* attributable to body 2. Such frame-dependent idiosyncrasies can be circumvented using the frame-invariant quantity $w_{\text{ext}} \equiv W_{\text{ext}} - \Delta K$, an *external* "work" defined using displacements measured relative to the system's CM.[5,7] Then Eq. (9) becomes the frame-invariant equation, $w_{\text{ext}} = \Delta U$. Choosing the system to be body 1 in our mirror symmetric collision, and using the $u = 0$ frame for convenience, $W_{\text{ext},1} = 0$ and $w_{\text{ext},1} = -\Delta K_1(u) = (1/2)mv^2 = \Delta U_1$. Although Newton's laws do not hold in the (noninertial) frame attached to the CM of body 1, $w_{\text{ext},1}$ is, formally, the external work done on body 1 in this frame.

If the mirror symmetry is broken—e.g., when the two bodies have different masses and elastic properties, result 4 assures that a zero-work frame exists. A simple example, for which the collision forces are purely repulsive and the contact surface is a plane perpendicular to the direction of motion, illustrates how this might occur. There are u frames, moving parallel to the collision dimension, in which the contact surface experiences both negative and positive displacements during the collision; i.e., both positive and negative work is done on each body during the collision. For a carefully chosen value of u, $W_{\text{ext},1} = 0$. Because $W_{\text{ext},1} = -W_{\text{ext},2}$ in *any* frame (see the Appendix), it is also true that $W_{\text{ext},2} = 0$, completing this example.

The frame-invariant equation $w_{\text{ext}} = \Delta U$ can be generalized to include heat transfer, which we view as a form of "invisible" external work, Q at a surface:

$$w_{\text{ext}} + Q = \Delta U. \tag{18}$$

This is an invariant form[5] of the first law of thermodynamics.[23] Thus our head-on inelastic collision, along with considerations of frame invariance and symmetry, creates a setting that leads naturally to the first law of thermodynamics.

The *postulated* internal energy function U, is fundamentally different from the quantities in the original work-energy theorem because it *hides* information. When bulk kinetic energy of a body is transformed into internal energy, macroscopically observable energy gets distributed among molecules in a way that is unknowable to a macroscopic observer. A partially inelastic collision loses less information than a completely inelastic one, and a fully elastic collision loses zero information. Information loss can be related to dissipation via entropy and the second law of thermodynamics. Rothstein[24] summed up the first and second laws of thermodynamics via two simple statements that are *apropos* here: (a) the conservation of energy, and (b) the existence of modes of energy transfer incapable of mechanical description. Our head-on collision provides an example of both, with (b) helping to explain the phenomenon of irreversible dissipation.

The many efforts in the literature devoted to clarifying work, heat, and energy in general illustrate that these topics are important but unfortunately unsettled for many physics teachers. The symmetric head-on collision used here has provided a new approach and provocative results that are potentially useful in clarifying work, introducing the concept of internal energy, and sharpening the distinction between internal energy and heat. The picture developed also explains in part the meaning of dissipation for inelastic collisions. At the very least, this approach illustrates the incompleteness and inadequacy of the simple statement, "the kinetic energy lost in an inelastic collision gets dissipated as heat." The latter sentence, though not strictly wrong, misses a host of interesting and potentially valuable physics. That physics is the essence of this article.

APPENDIX: A GENERALIZED TWO-BODY COLLISION

In the body of this paper we analyzed a *highly symmetric* collision to focus attention on the fact that a purely mechanical analysis is inadequate for inelastic collisions. We showed that internal energy changes are frame-independent, and, therefore, *truly* "internal." Here we show how the arguments for that simple, pedagogically attractive, case can be generalized.

Consider the general case of a collision between bodies with masses m_1 and m_2. The collision begins at the moment of first contact, is mediated by interaction forces across a time-varying contact surface, and ends at time τ, the moment of last contact or, in the case of bodies that stick together, the time at which an arbitrary end-of-collision criterion is satisfied. We adopt a vector-oriented analysis, carried out in an arbitrary reference frame moving with velocity $\mathbf{u}$ relative to the CM frame. The "velocity" of each body is the velocity of its *own* center-of-mass relative to the chosen frame of reference. Frame-dependent quantities are denoted by the argument $(\mathbf{u})$; the CM frame values of such quantities are labeled with the argument $(\mathbf{0})$. The initial and final velocities of the bodies, $\mathbf{v}_{1i}$, $\mathbf{v}_{2i}$, $\mathbf{v}_{1f}$, and $\mathbf{v}_{2f}$, and the frame-independent relative velocities, $\mathbf{w}_i$ and $\mathbf{w}_f$, satisfy the identities:

$$\mathbf{v}_{1s}(\mathbf{u})=\mathbf{v}_{1s}(\mathbf{0})-\mathbf{u}, \quad \mathbf{v}_{2s}(\mathbf{u})=\mathbf{v}_{2s}(\mathbf{0})-\mathbf{u}, \tag{A1a}$$

and

$$\mathbf{w}_s\equiv\mathbf{v}_{1s}(\mathbf{u})-\mathbf{v}_{2s}(\mathbf{u})=\mathbf{v}_{1s}(\mathbf{0})-\mathbf{v}_{2s}(\mathbf{0}), \quad \text{for } s=i,f. \tag{A1b}$$

The frame-independent change in the relative velocity is

$$\Delta\mathbf{w}\equiv\mathbf{w}_f-\mathbf{w}_i. \tag{A1c}$$

In the CM frame, the total momentum is zero; i.e., $m_1\mathbf{v}_{1s}(\mathbf{0})+m_2\mathbf{v}_{2s}(\mathbf{0})=0$ for $s=i,f$. Introducing the reduced mass of the system, $\mu\equiv m_1m_2/(m_1+m_2)$, the zero momentum condition and Eqs. (A1) imply the following identities:

$$\mathbf{v}_{1s}(\mathbf{0})=(\mu/m_1)\mathbf{w}_s, \tag{A2a}$$

$$\mathbf{v}_{2s}(\mathbf{0})=-(\mu/m_2)\mathbf{w}_s, \quad \text{for } s=i,f, \tag{A2b}$$

$$\Delta K_1(\mathbf{u})\equiv\tfrac{1}{2}m_1\{[v_{1f}(\mathbf{u})]^2-[v_{1i}(\mathbf{u})]^2\} = \frac{1}{2}\frac{\mu^2}{m_1}(w_f^2-w_i^2)-\mu\mathbf{u}\cdot\Delta\mathbf{w}, \tag{A3a}$$

$$\Delta K_2(\mathbf{u})\equiv\tfrac{1}{2}m_2\{[v_{2f}(\mathbf{u})]^2-[v_{2i}(\mathbf{u})]^2\} = \frac{1}{2}\frac{\mu^2}{m_2}(w_f^2-w_i^2)+\mu\mathbf{u}\cdot\Delta\mathbf{w}, \tag{A3b}$$

and

$$\Delta K_{\text{sys}}\equiv\Delta K_1(\mathbf{u})+\Delta K_2(\mathbf{u})=\tfrac{1}{2}\mu(w_f^2-w_i^2). \tag{A3c}$$

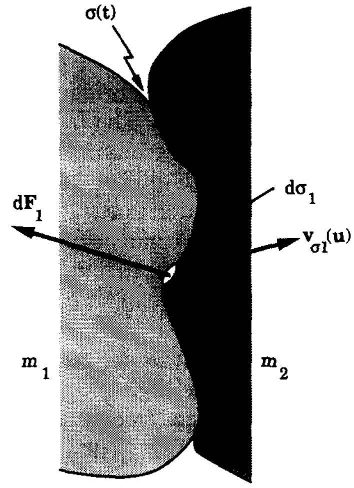

Fig. 3. The contact surface between two bodies during a collision. Here, $d\sigma_1$ is an element of the surface of body 1 that lies in the time-varying surface of contact $\sigma(t)$. $d\mathbf{F}_1$ is the force exerted on body 1 by body 2 across $d\sigma_1$ and $\mathbf{v}_{\sigma_1}$ is the instantaneous velocity of the surface element $d\sigma_1$.

Equations (A3) show that ΔK_{sys} *is frame-independent* even though the individual kinetic energy changes $\Delta K_1(\mathbf{u})$ and $\Delta K_2(\mathbf{u})$ are not. With some rearranging, Eqs. (A3a,b) can be rewritten as

$$\Delta K_r(\mathbf{u})=(\mu/m_r)\Delta K_{\text{sys}}+(-1)^r\mu\mathbf{u}\cdot\Delta\mathbf{w}, \quad \text{for } r=1,2. \tag{A4}$$

Notice that the frame-dependences of $\Delta K_1(\mathbf{u})$ and $\Delta K_2(\mathbf{u})$ are explicitly contained in the equal magnitude, but oppositely signed, final terms of Eqs. (A4).

Turning now to the works done on each body, we reiterate our assumption that the interaction between the two bodies occurs via contact forces. This implies that the interacting surface elements of each system experience identical displacements during the collision. As illustrated in Fig. 3, we consider a small surface element $d\sigma_1$ of body 1 that lies in the time-varying surface of contact $\sigma(t)$ between the two bodies. The force exerted on body 1 by body 2 across $d\sigma_1$ is $d\mathbf{F}_1$ and the displacement of $d\sigma_1$ during the period dt is $d\mathbf{s}(\mathbf{u})=\mathbf{v}_{\sigma_1}(\mathbf{u})dt=[\mathbf{v}_{\sigma_1}(\mathbf{0})-\mathbf{u}]dt$. To find the total work done on body 1 we integrate over the duration of the collision and over all elements of the contact surface to obtain

$$W_{\text{ext},1}(\mathbf{u})=\int_0^\tau\int_{\sigma(t)} d\mathbf{F}_1\cdot[\mathbf{v}_{\sigma_1}(\mathbf{0})-\mathbf{u}]dt = W_{\text{ext},1}(\mathbf{0})-\mu\mathbf{u}\cdot\Delta\mathbf{w}. \tag{A5a}$$

The second term follows from the fact that $m_1\Delta\mathbf{v}_1=\mu\,\Delta\mathbf{w}$ according to Eqs. (A1c) and (A2). Similar analysis for body 2 gives

$$W_{\text{ext},2}(\mathbf{u})=-W_{\text{ext},2}(\mathbf{0})+\mu\mathbf{u}\cdot\Delta\mathbf{w}. \tag{A5b}$$

Now we are in a position to assess the frame independence of the internal energy changes of bodies 1 and 2. Inserting Eqs. (A4) and (A5) into the extended work-energy theorem, Eq. (9), for each body gives

$$\Delta U_1(\mathbf{u}) = W_{ext,1}(\mathbf{0}) - (\mu/m_1)\Delta K_{sys}$$

and (A6)

$$\Delta U_2(\mathbf{u}) = W_{ext,2}(\mathbf{0}) - (\mu/m_2)\Delta K_{sys}.$$

Because the right sides of Eqs. (A6) are independent of the frame velocity $\mathbf{u}$, ΔU_1, *and* ΔU_2 *are frame independent.*

Conservation of energy for the two body system implies that $\Delta K_{sys} + \Delta U_1 + \Delta U_2 = 0$. Therefore, adding together Eqs. (A6) we find that

$$W_{ext,2}(\mathbf{0}) = -W_{ext,1}(\mathbf{0}), \tag{A7}$$

which, together with Eqs. (A5), establishes that $W_{ext,2}(\mathbf{u}) = -W_{ext,1}(\mathbf{u})$ for any value of $\mathbf{u}$.

Because $W_{ext,1}(\mathbf{0})$ defies evaluation without complete specification of the collision details, we will be content to treat it as a fundamental parameter of the collision. Equations (A4)–(A7) show that a knowledge of three such fundamental and frame-independent parameters—$\Delta\mathbf{w}$, ΔK_{sys}, and $W_{ext,1}(\mathbf{0})$—suffice to determine the changes in the kinetic and internal energies and the works done on both bodies in *any* specified reference frame.

Finally, we observe that even in the most general two-body collision, as long as the momenta of the bodies are altered, reference frames can *always* be found in which the work done by each body on the other during the collision is zero. Equations (A5) and (A7) show that this is the case for any reference frame with $\mathbf{u} = \mathbf{u}^*$ such that

$$\mathbf{u}^* \cdot \Delta\mathbf{w} = \mu^{-1} W_{ext,1}(\mathbf{0}). \tag{A8}$$

Equation (A8) specifies a family of reference frames (relative to the CM frame), each member of which has the *same* velocity component, $u^*_{\parallel} = W_{ext,1}(\mathbf{0})/(\mu|\Delta\mathbf{w}|)$, parallel to $\Delta\mathbf{w}$.

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WICKED SCIENTISTS, WICKED PHILOSOPHERS,...

[I shall not] waste time by defending science as a whole or scientists generally against a charge of inner or essential malevolence. The Wicked Scientist is not to be taken seriously: Dr. Strangelove, Dr. Moreau, Dr. Moriarty, Dr. Mabuse, Dr. Frankenstein (an honorary degree, this) and the rest of them are puppets of Gothic fiction. Scientists, on the whole, are amiable and well-meaning creatures. There must be very few wicked scientists. There are, however, plenty of wicked philosophers, wicked priests and wicked politicians.

Peter Medawar, *Pluto's Republic* (Oxford, New York, 1984), p. 311.