Energy and the Confused Student II: Systems

John W. Jewett Jr., California State Polytechnic University, Pomona, CA

Energy is a critical concept in physics problem-solving but is often a major source of confusion for students if the presentation is not carefully crafted by the instructor or the textbook. The first article[1] in this series discussed student confusion generated by traditional treatments of work. In any discussion of work, it is important to state that work is done on a *system* by a force. This phrasing has two important components: (1) the identification of the force that is doing the work and (2) the identification of the recipient of the work as a *system*. Very few textbook or lecture presentations use a system-based approach when performing an energy problem. The first two steps in approaching any energy problem should be:

1) Identify the system
2) Categorize the system

Identify the System

A *system* can be any of the following:

- A single object
- Two interacting objects
- A collection of several interacting objects
- A deformable object, such as a rubber ball or a sample of gas molecules
- A rotating object, such as a wheel
- A region of space, possibly deformable, such as the volume of an automobile engine cylinder above the piston

In the early part of the study of energy in a typical introductory physics course, the system of interest is often just a single object. In a system-based approach to teaching energy, however, the notion of a system is stressed even in these early discussions to prepare students for more complex situations that will be addressed soon.

Whatever form the system takes, there is a closed system boundary that surrounds the system and separates the system from everything outside, which is the *environment* or the *surroundings*. The system boundary may coincide with a physical surface, such as the outside surface of a baseball, but this coincidence is not necessary.

As an example, consider the relatively simple case of an object dragged across a surface with friction by a force **F** that is parallel to the surface. Suppose the student is asked to analyze this situation in terms of energy while the object is being dragged at a slow constant speed. In a traditional non-system-based approach, the student tends to focus on the object, because that is the only recipient of work that has been discussed. He or she is then likely to apply the work-kinetic energy theorem, $W = \Delta K$, to the object, because that is the only energy principle that has been discussed. This approach has four major flaws. First, as discussed in the first article in this series, the work done by the friction force on the object cannot be calculated because the displacement of the object is not the same as the displacement of the many points of application of the friction force. Second, the change ΔK in kinetic energy is zero because the object is dragged at constant speed. Third, there is likely to be a transfer of energy between the object and the surface by heat, which

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cannot be calculated and is not addressed in the work-kinetic energy theorem. Finally, the work-kinetic energy theorem does not contain a term for internal energy, which is an important component of the energy storage in this problem.

In comparison, a student familiar with a system-based approach to energy problems and the global nature of energy[2] would realize these difficulties and know that it is more fruitful to choose the system as the object *and* the surface, with a system boundary including the object and surface but not the agent applying the force **F**. (Because of the possibility of energy transfer by heat from the surface into the body possessing the surface, the system boundary should include the entire body, not just the zero-thickness two-dimensional surface.) In this case, the only transfer of energy into the system is the work done by the applied force **F** on the system, and the only change in energy for the system is a change in internal energy due to the friction:

$$W_{\mathbf{F}} = \Delta E_{\text{int}}. \tag{1}$$

There is indeed an exchange of energy by heat between the object and the surface; this exchange, however, is *within* the system. We have no way of knowing the individual changes in internal energy of the object and surface without further information. The equation above expresses all that we can know about this situation without this information.

Categorize the System

Once the system has been identified, it is important to determine whether the system is *isolated* or *non-isolated*. An isolated system is one for which there are no transfers of energy across the system boundary. A non-isolated system experiences transfers of energy across the boundary by one or more mechanisms.

In the previously discussed case of dragging the object across the surface, suppose we identify the object as the system. This system is clearly non-isolated because energy is crossing the boundary by work done by the applied force on the system and by heat: friction causes the object to become warm, so energy flows from the warm object to the air and into cooler parts of the surface that are encountered as the object moves. Further complicating the situation is the transfer of energy by mechanical waves—sound—as the object scrapes over the rough surface. If we identify the object and the surface as the system, this system is still non-isolated—work is done by the applied force on the system and energy transfers into the air by heat and sound. In order to identify an isolated system in this situation, we would need to include the air and the agent applying the force **F** so that work, heat, and sound represent transfers of energy within the system and not across the system boundary.

Physical situations involving the possibility of transfers of energy by heat, sound, and light are generally complicated because energy tends to spread over large distances by these processes. Many times, these transfers are neglected in order to make an approximation using a reasonably sized system. For example, for a falling object, air resistance is usually neglected so that the warming of the object due to the drag force and the transfer of energy by heat between the object and the air are ignored. In this case, if the system is identified as the object alone, the system is non-isolated due to the work done on the system by the gravitational force. If we identify the system as the object and the Earth, the system is isolated—there are no transfers of energy across the boundary of this system.

Once the system has been identified and categorized, the conservation of energy principle is applied to the system. The same principle is applied to isolated and non-isolated systems. This process is discussed further in the fourth article in this series.[2]

Internal and External Work

As another example of the importance of identifying the system, consider a common textbook[3] or lecture statement about potential energy:

> When a conservative force does work W, the potential energy corresponding to the force changes according to
>
> $$W = -\Delta U. \tag{2}$$

Such a statement makes no reference to the system, no reference to whether the conservative force is external or internal, and no reference to whether the work is done *on* or *within* the system. The bright student will recognize a contradiction: "If I lift a book to a higher shelf, I do positive work on the book-Earth system and there is an *increase* in gravitational potential energy of the system, not a *decrease*." The following equation can be written for this situation:

$$W = \Delta U, \tag{3}$$

where U represents the gravitational potential energy of the book-Earth system. The student looking at Eqs. (2) and (3) is likely to be confused about the minus sign that appears in one equation and not the other. It is critical in this case to discuss with students that *the works on the left-hand sides of the equations are not the same.* In Eq. (3), work W is the work done *on* a system by the surroundings and represents energy crossing the boundary of the system. Therefore, we can consider this to be *external* work because it represents an influence from outside the system.

In Eq. (2), work W is the work done *internal* to a system by one member of the system on another. In the case of a falling book described by Eq. (2), W is the work done by the gravitational force exerted on the book by the Earth, internal to the book-Earth system. It is strongly advised that different symbols[4] be used for the works in Eqs. (2) and (3) to emphasize the difference between them.

Equation (3) appears in fewer presentations than Eq. (2) and in very few cases is the comparison between Eqs. (2) and (3) made. Equation (3), however, is important because it is analogous to the work-kinetic energy theorem, $W = \Delta K$. In the work-kinetic energy theorem, energy is transferred into a system by work and the result is an increase in the kinetic energy of the system. Equation (3) represents an analogous situation in which energy is transferred into a system by work and the result is an increase in the potential energy of the system.

When relating conservative forces to potential energy, it is important to point out that the conservative force acts *between members of the system and the work done is within the system*. A complete and better statement than that associated with Eq. (2) above is as follows:

> Consider a system in which a conservative force acts between members of the system. If one member of the system moves so that the point of application of the conservative force undergoes a displacement and work W_c is done on it within the system by the force, the corresponding potential energy of the system changes according to
>
> $$W_c = -\Delta U. \tag{4}$$

Barrow[5] states, "The term 'work' can be recognized as just a crutch that paves the way for the later introduction of potential energy. . . . In all later mechanics problems, 'work' is discarded and potential and kinetic energies are used." This statement seems to confuse external and internal work. While internal work within a system is indeed related to a change in potential energy of the system, external work can be associated with a change in any type of energy in the system—kinetic (the work-kinetic energy theorem), potential (lift a book to a higher shelf), or internal (rub your hands together). Therefore, in the system-based approach to energy, work is definitely not "discarded" but rather an important distinction is made between internal work and external work on a system.

Multiple Systems

A given problem may involve different systems for different parts of the solution. For example, consider the common spring-gun ballistic pendulum experiment performed in many introductory laboratories.[6] The analysis of this apparatus involves *three* different systems. The first system is the spring and the projectile that is launched from the spring gun. This isolated system can be used to relate the speed of the projectile to the compression of the spring by conservation of mechanical energy. The second system is the projectile and the pendulum arm. Conservation of momentum is applied to this isolated system to determine the relationship between the initial speed of the projectile and the final speed of the projectile-arm combination. Finally, conservation of mechanical energy is used for the isolated system of the projectile, the pendulum arm, and the Earth to relate the final speed of the projectile-arm combination to the final height of the center of mass of the combination.

Keeports[7] discusses the situation of a helium balloon, describing the surprise his students express when they realize that potential energy decreases as the balloon moves upward in the air. In this discussion, the word *system* is never used and the potential energy is always described as that "of the balloon."[8] Indeed, his Eq. (6), $U = (mg - \rho Vg)y$ supposedly expresses the potential energy "of the balloon" as a combination of that associated with the gravitational force and that associated with the buoyant force. The equation, however, mixes two different systems. The potential energy associated with the gravitational force belongs to the system of the balloon and the Earth. The potential energy associated with the buoyant force belongs to the system of the balloon and the air. If the discussion were to define the system as the balloon, the Earth, and the air, the equation might be justified. For this system, however, the net force between the balloon and the Earth-air combination is a repulsive force. The system can be modeled as an isolated system in which there are two components that repel one another. Therefore, the decrease in potential energy for the system as the balloon rises is no more surprising than that of a system of two protons moving apart because of the repulsive force between them. Rather than presenting this example as a surprise, it would be valuable to recognize the opportunity for modeling and present the discussion above to show the parallels between two different systems (proton and proton; Earth-air combination and balloon) that can both be analyzed as a pair of repelling objects.

Problems

Consider the following two problems:

1.) A ball of mass m is dropped from a height h above the surface of the Earth (Fig. 1) and air resistance is neglected. With what speed does it strike the Earth?

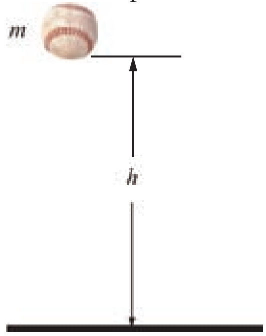

Fig. 1. A ball of mass *m* is dropped from a height *h*. How fast is it moving when it strikes the Earth?

2.) Each of four small spheres has mass m. Between each pair of spheres is a compressed spring, with the springs forming a square of side h (Fig. 2). The springs are identical, have force constant k and negligible mass, and are not fastened to the spheres. The natural length of each spring is L. The spheres are tied with light strings that pass through the axes of the four springs. The entire apparatus is in a gravity-free region of space. All four strings are simultaneously cut so that the spheres are pushed away by the springs and fly away. With what speed are the spheres traveling when they are no longer in contact with the springs?

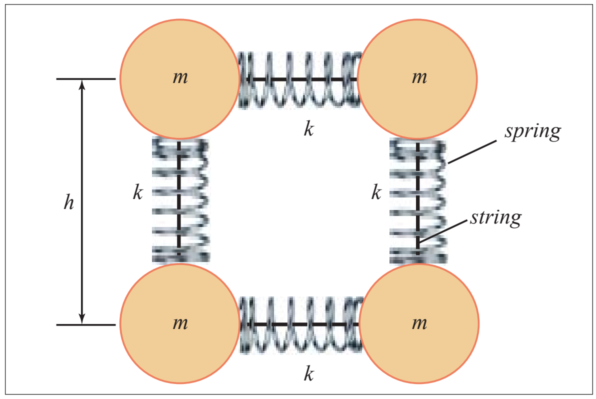

Fig. 2. Four spheres of mass *m* are connected in a square of side *h* by strings. Identical springs are compressed between each pair of spheres. When the strings are cut, the spheres fly away. How fast are the spheres moving when they leave the springs?

Are these problems fundamentally different? The student who is not familiar with the system-based approach may be able to get away with the common but inappropriate technique of "set mgh equal to $1/2mv^2$" to solve the first problem but may be stymied by the second. The second problem involves multiple kinetic energies and multiple potential energies; the solution requires familiarity with the energy of a system rather than simply with the energy of a single object.

The student who has learned the system-based approach will use the same beginning steps for both problems. For problem 1, we identify the system as the ball and the Earth. This is an isolated system in which no non-conservative forces act, so we write an equation for conservation of mechanical energy:

$$\Delta K + \Delta U_g = 0, \quad (5)$$

where U_g refers to gravitational potential energy.

For problem 2, we identify the system as the four spheres and the four springs. This is also an isolated system in which no non-conservative forces act, so we write an equation for conservation of mechanical energy:

$$\Delta K + \Delta U_s = 0, \tag{6}$$

where U_s refers to spring (elastic) potential energy. The approaches for both problems are the same; only the evaluations of the energies differ. These problems will be solved in full in the fourth article[2] in this series.

In light of these discussions, consider the following true-false question related to the earlier discussion of a block moving on a surface:

> **True or False?** An object is dragged across a tabletop at constant velocity by an applied force that is parallel to the surface. Because the object is in equilibrium, the friction force is equal in magnitude to the applied force. Therefore, the work done on the object by the friction force is equal in magnitude to that done by the applied force. The net work done on the object by all forces is zero.

This argument is tempting to many students but, as discussed by Sherwood and Bernard[9] and Chabay and Sherwood,[10] it is false. This can be argued from two points of view. The first relates to the definition of work discussed in the first article in this series.[1]

Although the applied force and the friction force have equal magnitudes, the displacement of the applied force is not the same as the many displacements of the friction force at a large number of contact points. Therefore, the works done by the two forces are not the same in magnitude and do not cancel.

The second point of view relates to a careful system analysis of energy. Let us identify the object as the system. If we claim that the net work done on the object by all forces is zero, and there are no other transfers of energy into the system, then the energy of the system must remain fixed. The kinetic energy of the system indeed remains fixed because the object moves at constant speed. But, from common experience, we know that dragging an object over a surface causes the object to become warmer—its internal energy increases. If zero work were done on the system of the object, there would be no source for this increased internal energy.

Conclusion

We have presented several cases in which it is important to identify the system of interest when addressing a problem with an energy approach. Failure to do so can lead to errors and misconceptions. As physics teachers, we have a duty to convince our students of the importance of identifying and categorizing the system when using an energy approach to solve a problem. In the next installment of this series, we will discuss confusion generated by the careless use of language when discussing energy.

References

1. J.W. Jewett, "Energy and the confused student I: Work," *Phys. Teach*. **46**, 38–43 (Jan. 2008).
2. J.W. Jewett, "Energy and the confused student IV: A global approach to energy," *Phys. Teach.*, to be published in April 2008.
3. As a textbook author myself, I do not specifically identify problematic statements in other authors' textbooks in this series of articles. I do not want this series to appear as a marketing tool but rather as a professional communication that offers a set of suggestions for improving the teaching of energy to our students. I present items from several textbooks in general terms and not as direct quotes.
4. See, for example, R.A. Serway and J.W. Jewett, *Physics for Scientists and Engineers*, 7th ed. (Brooks/Cole, Belmont, CA, 2008), Chap. 7, in which W is used for Eq. (3) and W_c (subscript c for *conservative* force) for Eq. (2). See also R.W. Chabay and B.A. Sherwood, *Matter & Interactions I: Modern Mechanics,* 2nd ed. (Wiley, Hoboken, NJ, 2007), Chap. 5, in which W_{surr} (*surroundings*) is used for Eq. (3) and W_{int} (*internal*) for Eq. (2).
5. G.M. Barrow, "Thermodynamics should be built on energy—not on heat and work," *J. Chem. Educ.* **65**(2), 122–125 (Feb. 1988).
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7. D. Keeports, "How does the potential energy of a rising helium-filled balloon change?" *Phys. Teach*. **40,** 164-165 (March 2002).
8. This misleading phrasing for potential energy will be discussed in J.W. Jewett, "Energy and the confused student III: Language," *Phys. Teach*., to be published in March 2008.
9. B.A. Sherwood and W.H. Bernard, "Work and heat transfer in the presence of sliding friction," *Am. J. Phys.* **52,** 1001–1007 (Nov. 1984).
10. R.W. Chabay and B.A. Sherwood, *Matter & Interactions I: Modern Mechanics,* 2nd ed. (Wiley, Hoboken, NJ, 2007), pp. 291–293.

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John W. Jewett Jr. *is professor emeritus at California State Polytechnic University. He has authored* The World of Physics; Mysteries, Magic, and Myth *and co-authored* Physics for Scientists and Engineers*, seventh edition, and* Principles of Physics*, fourth edition. He won the AAPT Excellence in Undergraduate Physics Teaching Award in 1998;* **jwjewett@csupomona.edu**