Energy and the Confused Student IV: A Global Approach to Energy

John W. Jewett Jr., California State Polytechnic University, Pomona, CA

Energy is a critical concept in physics problem-solving, but is often a major source of confusion for students if the presentation is not carefully crafted by the instructor or the textbook. In the first three articles[1-3] in this series we discussed several issues related to the teaching of energy concepts. We have saved a major single issue for this article: the presentation of energy by means of a global approach. Energy, energy transfers, and energy transformations are at the heart of *every* process that occurs in physics, chemistry, biology, astronomy, and geology. Consequently, it is useful and highly instructive to discuss this global nature of energy from the very beginning, when energy is first introduced in mechanics.

A Typical Traditional Approach

Unfortunately, energy is often presented in textbooks and classrooms in a disjointed manner such that students may believe that there are several fundamental energy equations. Despite having multiple energy equations available, however, students taught in this fashion cannot write simple energy equations that describe the operation of everyday systems such as their stereo system, a lawn mower, or a light bulb. With the approach described in this article, students have *one* fundamental energy equation and can write appropriate energy equations for these sample systems, as well as many more.

A typical traditional approach introduces the work-kinetic energy theorem when discussing moving objects. Things quickly become more complicated, however, when friction is introduced. As discussed in the first article[1] in this series, we cannot calculate the work done by friction because the displacement of the object is not the same as the displacements of the many points of application of the friction force, so the work-kinetic energy theorem is of no use. Deformable and rotating objects are often avoided in the classroom or textbook because they offer complications that cannot be handled by a traditional approach to work and the work-kinetic energy theorem.

When potential energy is discussed, the conservation of mechanical energy equation is introduced. Because systems are often not emphasized in a traditional approach, as discussed in the second article[2] in this series, students may believe that the conservation of mechanical energy equation is separate from the work-kinetic energy theorem—another equation that involves energy, but seemingly a separate idea.

Finally, when thermodynamics is discussed, internal energy and heat are introduced. At that time, a third apparently disconnected energy equation is introduced—the first law of thermodynamics.

This disjointed approach is reminiscent of the historical growth of thermodynamics as a separate topic from mechanics. These areas of physics were unified a long time ago. It's time we teach a unified, global approach in the classroom!

The Global Approach

It is my position in this article that there is only *one* fundamental energy equation and that all other energy equations are special cases. The fundamental

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equation is called the *conservation of energy equation*[4] or the *continuity equation for energy,* both of which can be abbreviated as CEE:

$$\Delta E_{\text{system}} = \sum T, \tag{1}$$

where T represents the amount of energy transferred (T for *transfer*) across the boundary of the identified system by a given mechanism. The general conceptual basis of the equation is this: the only way the total energy E_{system} of a system can change is if energy crosses the system boundary by one or more mechanisms described by T. The mathematical basis is this: the total change in energy of the system during some time interval is exactly equal to the net amount of energy crossing the system boundary. The summation sign indicates that energy may cross the boundary by several methods. It is instructive to compare this to a student's bank account—the balance does not change if there are no transfers into or out of the bank system. When there are transfers in the form of deposits, withdrawals, fees, interest, and checks written, however, the balance changes by exactly the net amount of money transferred by these processes.

In my teaching of classical physics, the expanded version of Eq. (1) is expressed as follows:[5]

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{\text{MT}} + T_{\text{MW}} + T_{\text{ER}} + T_{\text{ET}}. \tag{2}$$

The left-hand side of this equation shows three ways of storing energy in the system: kinetic energy K, potential energy U, and internal energy[6] E_{int}. The change in the total energy stored in the system is found by adding the three individual changes for these types of energy storage.

The kinetic energy K on the left side of the CEE, Eq. (2), is the sum of the translational kinetic energy of the center of mass of the system, rotational kinetic energy around the center of mass of the system, and any kinetic energy associated with radial motions of the members of the system with respect to the center of mass. The potential energy U includes all types, such as gravitational, electric, and elastic. In addition, I include here chemical potential energy of fuel or explosives, and biological potential energy resulting from eaten meals. The internal energy E_{int} includes the energy associated with randomized motion of molecules, measured by temperature, and bond energies between molecules, associated with the phase (solid, liquid, or gas) of the system.

On the right side of the CEE is the total amount of energy that crosses the boundary of the system, expressed as the sum of the energy transferred by six common processes:

W: *work* done on the system by external forces whose points of application move through displacements

Q: energy transferred across the boundary of the system by *heat* due to a temperature difference between the system and its environment

T_{MT}: energy transferred across the boundary of the system by *matter transfer* (such as transferring a fuel into a tank)

T_{MW}: energy transferred across the boundary of a system by *mechanical waves* such as sound waves or seismic waves

T_{ER}: energy transferred across the boundary of a system by *electromagnetic radiation* such as light or microwaves

T_{ET}: energy transferred across the boundary of a system by *electrical transmission* from a battery or other electrical source

It is instructive to spend time discussing this equation and its individual terms when energy is first introduced. Students are familiar enough from everyday life with the six types of energy transfer in the equation that they can quickly understand the nature of energy transfers and the meaning of the equation. In my experience, early in the course, many students begin with the full Eq. (2) and cross off terms that do not apply to a given situation. After gaining more familiarity and experience with the approach, they often begin with Eq. (1) and build the appropriate equation by listing just those terms that are needed to analyze the situation.

In using the CEE, it is important to identify the system of interest as well as the time interval of interest. For some systems, different choices of time intervals may result in a different CEE. For example, if the system is the set of coils in a toaster, there is a change in the internal energy E_{int} during a time interval just after the toaster is turned on, but no such change for a later time interval during which the temperature has stabilized.

The discussion of the various means of energy transfer in association with the conservation of energy equation addresses another weakness of traditional textbook[7] and classroom discussions related to *power*. Because work is often the only means of energy transfer to be discussed in a traditional coverage of mechanics, it is often stated that "power is the rate at which work is done." This is an incomplete statement and will leave students wondering about power ratings on their light bulbs and stereo systems. When all forms of energy transfer are discussed in a global approach to energy, however, the correct statement about power can be made:[8] "power is the rate at which energy is transferred across the boundary of the system." Students taught by this approach will understand what a 60-watt light bulb is—as well as what the lumen rating on the light bulb package represents. They will also understand sound power from their stereo speakers and light power coming from the Sun.

Arons[9] supports the view that a global approach to energy for all processes, as represented by the CEE, is advisable: "It seems a shame . . . to give students so narrow and restricted a view of the concept of energy and its conservation . . . . Furthermore, recognizing what happens phenomenologically in everyday experiences such as running, jumping, accelerating a car, and confronting frictional effects, gives personal relevance, richness, and greater meaning to the physical concepts not attainable in the restricted development."

As discussed in the third article[3] in this series, it is important to distinguish between transfers of energy across the boundary of the system and transformations of energy within the system. I identify three primary types of transformation mechanisms: (1) work, (2) chemical reactions, and (3) nuclear reactions. The work in this context is not the W term on the right-hand side of the CEE, Eq. (2)—work in that equation is energy transfer across the boundary of the system. Work as a transformation mechanism is *internal* work as discussed in the first article[1] in this series. It is work done by one system component on another, causing a transformation of energy. For example, within a ball-Earth system, work done by the gravitational force between the Earth and the ball causes a transformation from gravitational potential energy to kinetic energy.

Chemical reactions cause transformation of potential energy in a system of chemicals to other forms, such as internal energy and possibly kinetic energy of flying pieces in an explosion. Nuclear reactions transform energy stored in the nucleus into kinetic energy of outgoing particles and internal energy of material surrounding the reaction. In general, *transformation of energy causes a conversion of one type of storage of energy in the system into another type*.

It is also possible to discuss transfers of energy *within* the system that redistribute the energy but do not change the total energy in the system. For example, a system may include a hot object and a cold object. In this case, there may be a transfer of energy by heat and electromagnetic radiation from the hot object to the cold object. As long as energy is only transferring between members of the system, this is an isolated system and the right-hand side of the CEE is zero. The internal transfers are redistributing the energy so that the cold object is gaining internal energy while the hot object is losing internal energy, but the total energy of the system remains constant. A process such as this would not be evident in the conservation of energy equation for the system, but the process might be described by other principles, such as requiring thermal equilibrium for the system components. *Transfers of energy within the system often do not cause a conversion of one type of storage of energy in the system into another type—the energy is redistributed among the system components but remains in the same form.*

Categorize the System

The process of categorization of the system was discussed in the second article[2] in the series. In that discussion, we identified a *non-isolated system* as one for which there are transfers of energy across the boundary of the system by at least one mechanism. In this case, the CEE for the system is represented by Eq. (1). An *isolated system* is one for which there are no transfers of energy across the boundary of the system by any mechanism. The CEE for the system in this case reduces to the special form,

$$\Delta E_{\text{system}} = 0 \,. \tag{3}$$

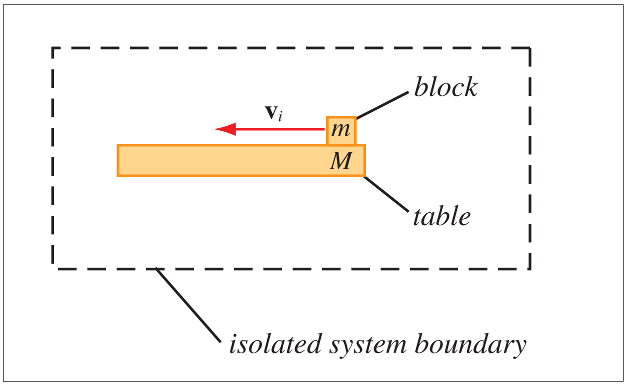

Fig. 1. A block of mass *m* is set sliding across a table of mass *M*. The table is free to move on a frictionless surface and the system of the block and table is isolated.

In a process involving an isolated system, only transformations of energy occur, such that the system energy remains fixed. (There may also be transfers of energy within the system, but these will generally not affect the CEE.)

Another categorization possibility that was not discussed previously is the *non-isolated system in steady state*. This system is described by a CEE as follows:

$$0 = \sum T. \tag{4}$$

In this system, energy is transferring across the boundary of the system, but the rate of transfer of energy into the system balances the rate of transfer out of the system, so that the total energy in the system remains fixed. The Earth is an example of a non-isolated system that is approximately in steady state. The rate of energy transferring into the system by visible light from the Sun is balanced with the rate of energy transfer out of the system by infrared radiation. The global warming problem is related to the fact that the Earth is not *quite* in steady state. Because of the capturing of infrared radiation by greenhouse gases, the surface temperature of the Earth is slowly increasing.

Mungan[10] provides a problem in which a block slides over a table that is free to move in a chamber designed to allow no energy to enter or leave the system. Figure 1 shows this situation. At time $t = 0$, the table is at rest and the block is sliding with speed v_i. Eventually, the block and table move with a common speed v_f.

Mungan's discussion of this problem is designed to show that it can be solved without introducing the concepts of work or heat. Using the global approach to energy, the student would not think of introducing work or heat as energy transfers across the system boundary because the system of the block and table is isolated. Work *internal* to the system[2] does indeed occur and contributes to changes in the speeds of the block and table. Additionally, the system gains internal energy due to the friction force between the block and the table,[1] which is distributed between the block and the table by heat. Therefore, work and heat are occurring within the system, but that is not what is on the right-hand side of the CEE. The terms W and Q that appear in the CEE represent transfers of energy across the boundary of the system and are zero for this situation. The work done within the system is a transformation process that converts kinetic energy to internal energy, and the process of heat that occurs simply distributes internal energy between the components of the system.

Mungan also notes that it is tempting [for students] to identify W with ΔK and Q with ΔU. From the global energy approach, the student will not be tempted to make this identification because he or she has been taught (and hopefully has learned) that any type of energy transfer can cause a change in any type of energy storage.

Reducing the Conservation of Energy Equation

Let us consider three special cases to see how to use the conservation of energy equation in practice. First, suppose the system is a single object that can be modeled as a particle and the time interval is one during which a single force acts on the object in empty space. Because of the influence of the external force, this system is non-isolated. The only type of energy that can change in the system is kinetic energy, and the only way energy is transferring into the system is by work. Equation (2) in this case reduces to

$$\Delta K = W, \tag{5}$$

which is the work-kinetic energy theorem. Therefore, this equation that is so often interpreted by students as fundamental is seen to be a special case of the general conservation of energy equation.

Next, consider an arbitrary system that is isolated so that there are no transfers of energy into or out of the system. Suppose also that no nonconservative forces act within the system. Then there is no conversion of mechanical energy to internal energy and Eq. (2) becomes

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad K_f + U_f = K_i + U_i , \qquad (6)$$

which is the familiar expression for conservation of mechanical energy for an isolated system. For example, for a falling ball, the system is chosen as the ball and the Earth and the potential energy is gravitational.

Finally, consider a system of an ideal gas in a stationary cylinder. The piston in the cylinder can be moved so that work can be done on the gas. The walls of the cylinder are thermally conducting, so that energy can enter or leave the system by heat. In this case, Eq. (2) becomes

$$\Delta E_{\text{int}} = W + Q , \qquad (7)$$

which is the first law of thermodynamics. Therefore, all three of the "fundamental" energy equations mentioned earlier in this discussion can be generated from the true fundamental energy equation, the CEE.

As further evidence of the generality of the conservation of energy equation, consider the photoelectric effect. In this process, a photon strikes a clean metal surface and ejects an electron. Identify the system as the metal and the single electron that is ejected. This system is non-isolated because energy crosses the boundary of the system by electromagnetic radiation in the form of a photon. In this case, Eq. (2) reduces to

$$\Delta K + \Delta U = T_{\text{ER}} , \qquad (8)$$

where T_{ER} is the transfer of energy into the system by the photon. The potential energy U is that of the metal-electron system. We identify the configuration of the electron outside the metal as having zero potential energy. When the electron is inside the metal, it is bound within the metal, which we can model by identifying the potential energy of the system as $-U_0$. Therefore the change in potential energy of the system is $\Delta U = 0 - (-U_0) = U_0$.

The kinetic energy K of the system is that of the ejected electron. We model its kinetic energy as zero when it is in the metal. (The single electron's kinetic energy makes a negligible contribution to the metal's internal energy when it is in the metal. We ignore this very small correction.) Calling the kinetic energy of the electron when it is ejected K, the change in kinetic energy of the system is $\Delta K = K - 0 = K$. Therefore, Eq. (8) for this process becomes

$$K + U_0 = T_{\text{ER}} . \qquad (9)$$

If we now recognize the energy transferred into the system by the photon as $T_{\text{ER}} = hf$, where h is Planck's constant and f is the frequency associated with the photon, we have

$$K + U_0 = hf , \qquad (10)$$

which is Einstein's equation for the photoelectric effect, with U_0 identified as the work function of the metal and K as the maximum kinetic energy of the ejected electron.

Let us now address the situations mentioned earlier that the student could not analyze after traditional instruction in energy. Consider a stereo system as the system of interest and choose a time interval from just before we turn it on to an instant after it has been on for a couple of minutes. The system is non-isolated. The appropriate reduction of Eq. (2) in this case is

$$\Delta E_{\text{int}} = Q + T_{\text{MW}} + T_{\text{ER}} + T_{\text{ET}} . \qquad (11)$$

The change in internal energy E_{int} on the left is signaled by the increasing temperature of the system as it operates from a cold start. Energy leaves the warm stereo by heat (Q) into the cool air and leaves the system from the speakers by sound waves (T_{MW}). Energy T_{ER} enters the system by electromagnetic radiation due to the input radio signal and leaves the system from various display lights. The largest input energy is by electrical transmission (T_{ET}) through the power cord.

Now consider a lawn mower as a system and the time interval to be from just before it is filled with gasoline to an instant after it has been operating for a few minutes. This system is also non-isolated. The appropriate reduction of Eq. (2) is

$$\Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MT} + T_{MW} . \quad (12)$$

The kinetic energy K corresponds to the rotating blade and motor parts and has increased because the mower was not operating at the beginning of the time interval. The potential energy U of the system is associated with the gasoline in the tank and has increased due to the filling of the tank. Because the mower is warming up, there is a change in internal energy E_{int}. The work W is done by the operator pulling the starter cord. Energy Q leaves by heat into the cool air, while other energy leaves the system by sound waves (T_{MW}). The term T_{MT} corresponds to the process of filling the tank with gasoline.

Finally, let's address a light bulb filled with a gas. We identify the filament of the light bulb as the system and the time interval as a one-minute interval long after the bulb has been turned on. The appropriate reduction of Eq. (2) is

$$0 = Q + T_{ER} + T_{ET} . \quad (13)$$

The filament is a non-isolated system in steady state. In this expression, energy T_{ET} enters the bulb filament by electrical transmission. Energy leaves the filament by heat (Q) into the gas in the bulb and by infrared and visible light (T_{ER}). If the light bulb is evacuated, the Q term vanishes.

Problems

Let us now address the two problems raised in the second article in this series.[2]

Problem 1

A ball of mass m is dropped from a height h above the surface of the Earth (Fig. 2) and air resistance is neglected. With what speed does it strike the Earth?

Choosing the system as the ball and the Earth and the time interval to be from when the ball is dropped to an instant just before it strikes the ground, Eq. (2) becomes

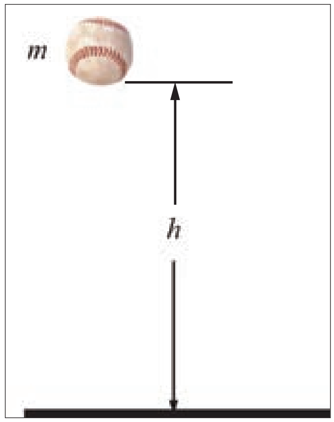

Fig. 2. A ball of mass *m* is dropped from a height *h*. How fast is it moving when it strikes the Earth?

$$\Delta K + \Delta U_g = 0. \quad (14)$$

We define the configuration of the system when the ball is at the ground to have zero gravitational potential energy. Therefore,

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(0 - mgh\right) = 0 \quad (15)$$
$$\rightarrow \quad v = \sqrt{2gh} .$$

This is clearly a simple problem; it is presented for purposes of comparison to Problem 2.

Problem 2

Each of four small spheres has mass m. Between each pair of spheres is a compressed spring, with the springs forming a square of side h (Fig. 3). The springs are identical, have force constant k and negligible mass, and are not fastened to the spheres. The natural length of each spring is L. The spheres are tied with light strings that pass through the axes of the four springs. The entire apparatus is in a gravity-free region of space. All four strings are simultaneously cut so that the spheres are pushed away by the springs and fly away. With what speed are the spheres traveling when they are no longer in contact with the springs?

We choose the system to be the four spheres and the four springs. The time interval is from before the strings are cut until any instant after the spheres lose contact with the springs. Eq. (2) becomes

$$\Delta K + \Delta U_s = 0. \quad (16)$$

The only difference between this problem and Problem 1 is that we need to consider the kinetic energy of four objects and the potential energy of four springs. Because of the symmetry of the situation, we know that all four spheres will have the same speed after the strings are cut. Because the

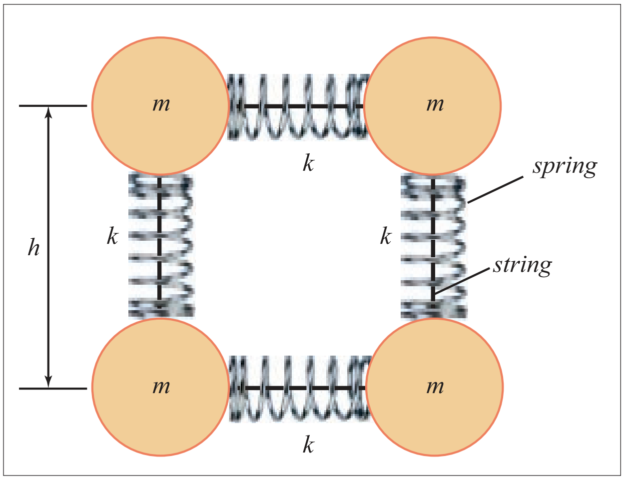

Fig. 3. Four spheres of mass *m* are connected in a square of side *h* by strings. Identical springs are compressed between each pair of spheres. When the strings are cut, the spheres fly away. How fast are the spheres moving when they leave the springs?

springs are identical, they will each store the same potential energy before the strings are cut. We define the configuration of the system when all the springs are uncompressed to have zero elastic potential energy. Therefore,

$$\left[4\left(\tfrac{1}{2}mv^2\right)-0\right]+\left\{0-4\left[\tfrac{1}{2}k\left(L-h\right)^2\right]\right\}=0$$
$$\rightarrow \quad v=\sqrt{\frac{k}{m}}\left(L-h\right). \tag{17}$$

Notice that the general approach to both problems is the same. A student who has been taught the global approach to energy is undaunted by the seeming complexity of the second problem. Furthermore, that student will be comfortable later on when he or she sees the following problem.

Problem 3

> Each of four small spheres has mass *m* and electric charge *q*. The spheres are tied with insulating strings so they form a square of side *h* in a gravity-free region. All four strings are simultaneously cut so that the spheres fly away. With what speed are the spheres traveling when they are infinitely far apart?

The student who has learned the global approach will recognize this as the same form of problem as Problems 1 and 2 and that it would be handled in exactly the same way. He or she will also recognize an important difference in this problem because of the nature of the electric force in the four-object arrangement compared to the spring force: We must add up electric potential energies for all binary pairs of interacting particles in the system:

$$\Delta K+\Delta U_e = 0 \tag{18}$$

$$\left[4\left(\tfrac{1}{2}mv^2\right)-0\right] + \left\{0-\left[4k_e\left(\frac{q^2}{h}\right)+2k_e\left(\frac{q^2}{\sqrt{2}h}\right)\right]\right\}=0 \tag{19}$$

$$\rightarrow \quad v=\sqrt{\frac{k_e q^2}{mh}\left(2+\frac{1}{\sqrt{2}}\right)}. \tag{20}$$

Conclusion

We reiterate the importance of energy in understanding physical processes and solving physics problems. Because of this importance, we should take the time early in the course to share with students the global nature of energy and a true fundamental energy equation that can be used in all situations. Furthermore, we should define work carefully, emphasize the importance of identifying the system, and make sure we observe careful use of the language. With these efforts on our parts, we should have students at the end of our courses who truly understand energy and can readily handle analyses of situations involving energy and energy transfers. In the final installment in this series, we show how to use the global approach to energy to solve several problems, including the two problems suggested by Mungan[11] that were discussed in the second article[2] in the series.

References

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2. J.W. Jewett, "Energy and the confused student II: Systems," *Phys. Teach.* **46**, 81–86 (Feb. 2008).
3. J.W. Jewett, "Energy and the confused student III: Language," *Phys. Teach.* **46**, 149–153 (March 2008).
4. The conservation of energy equation in a global

approach to energy is discussed in R.A. Serway and J.W. Jewett, *Physics for Scientists and Engineers,* 7th ed. (Brooks/Cole, Belmont CA, 2008), pp. 196–204.

5. Some nontraditional treatments of mechanics introduce relativity along with classical mechanics and include discussions of rest energy, such as in R.W. Chabay and B.A. Sherwood, *Matter & Interactions I: Modern Mechanics*, 2nd ed. (Wiley, Hoboken NJ, 2007). In this case, a term for changes in rest energy can be added to the left side of Eq. (2).
6. One complicating factor in the global approach to energy is that the symbol U is used in some traditional presentations to represent potential energy in mechanics as well as internal energy in later discussions of thermodynamics. In a global approach, where these two types of energy storage are discussed together, they must be distinguished from one another; hence the notations U for potential energy and E_{int} for internal energy.
7. As a textbook author myself, I do not specifically identify problematic statements in other authors' textbooks in this series of articles. I do not want this series to appear as a marketing tool, but rather as a professional communication that offers a set of suggestions for improving the teaching of energy to our students. I present items from several textbooks in general terms and not as direct quotes.
8. Some physicists also include the rate of transformation of energy within a system in the definition of power. For example, consider a person climbing a ladder. Identify the system as the person, the ladder, and the Earth. The notion of power in this situation could be applied to the rate at which potential energy in the body of the person from previous meals is transforming to gravitational potential energy as the person rises.
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John W. Jewett Jr. *is professor emeritus at California State Polytechnic University. He has authored* The World of Physics; Mysteries, Magic, and Myth *and co-authored* Physics for Scientists and Engineers*, seventh edition, and* Principles of Physics*, fourth edition. He won the AAPT Excellence in Undergraduate Physics Teaching Award in 1998;* **jwjewett@csupomona.edu**