
Energy and the Confused Student I: Work

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Energy is a critical concept that is used in analyzing physical phenomena and is often an essential starting point in physics problem-solving. It is a global concept that appears throughout the physics curriculum in mechanics, thermodynamics, electromagnetism, and modern physics. Energy is also at the heart of descriptions of processes in biology, chemistry, astronomy, and geology. Therefore, it is important to discuss the topic of energy clearly and effectively in textbook and lecture presentations. Unfortunately, this topic is filled with possibilities for student confusion if the presentation is not carefully crafted by the instructor or the textbook. There are a number of steps, however, that can be taken in teaching about energy that reduce or eliminate the sources of confusion for students.

This is the first in a series of five articles offering an approach to improving the discussions of energy in textbooks and classroom lectures, thereby enhancing the subsequent understanding of energy by students and their use of that understanding in solving problems. We begin by discussing the concept of *work*. An approach in some textbook and classroom solutions of more complicated problems is to define work and then introduce a number of work-like quantities (such as “pseudowork” or “center-of-mass work”) as well as a variety of energy-like equations (such as the “pseudowork-kinetic energy theorem” or the “center-of-mass equation”). The introduction of these multiple quantities and equations can be confusing for introductory students. This series of articles will argue that complicated problems can be solved with only *one* defini-

tion of work and *one* energy equation, without the necessity for introducing other work-like properties or energy-like equations.

Identifying the Displacement in the Definition of Work

The teaching of work has generated a number of discussions in the literature.¹⁻¹² Common textbook introductions of work involve a discussion of a force \mathbf{F} applied to an object, which then moves through some displacement $\Delta\mathbf{r}$. This is followed by the presentation of the following equation or a variation thereof:

$$W = \mathbf{F} \cdot \Delta\mathbf{r} = F \Delta r \cos \theta, \quad (1)$$

where θ is the angle between the force and displacement vectors. Many textbook discussions¹³ identify $\Delta\mathbf{r}$ as “the displacement of the object” or as simply “the displacement,” without identifying what is being displaced. This vagueness leads to conceptual difficulties later in the study of mechanics when the student encounters friction forces or forces applied to deformable or rotating objects.

In some approaches, the displacement in Eq. (1) is purposely left arbitrary so that it can be identified differently, depending on the work-like quantity that is being discussed. For example, one work-like quantity that is sometimes addressed is “pseudowork”²⁻⁴ or “center-of-mass work.”¹² For this work-like quantity, the displacement is that of the center of mass of the object or system upon which the work is done. In another approach by Chabay and Sherwood,⁶ an energy

principle is applied to a “point-particle system” represented by modeling a system as if all of its mass were at the center of mass. In this case, the displacement of interest is again that of the center of mass.

In my method for solving problems using an energy approach, the definition of $\Delta \mathbf{r}$ is *always* specified as *the displacement of the point of application of the force*. For a system consisting of a single particle or nondeformable, nonrotating object, the displacement of the point of application of the force is the same as the displacement of the center of mass of the object or system. Consequently, identifying $\Delta \mathbf{r}$ as the displacement of the object does not cause immediate trouble for these systems. However, for a deformable or rotating system, the displacement of the center of mass of the system can be different from the displacement of the point of application of the force. In this case, the textbook author or lecturer may introduce pseudowork or center-of-mass work. In this series of articles, I claim that this new work-like quantity is not necessary, and I discuss an energy approach that does not require it.

It is entirely *possible* to teach mechanics without specifying a single definition of the displacement in Eq. (1). This can be done in two ways:

1. Avoid any problems in mechanics involving anything other than a particle and then redefine the displacement $\Delta \mathbf{r}$ when teaching thermodynamics. But why put the students through the disadvantage of shortchanging their study of mechanics and then forcing them to forget the definition of displacement learned much earlier and replace it with a different one in thermodynamics? Indeed, as Chabay and Sherwood⁶ point out, “Often in traditional courses the concept of energy introduced to help solve mechanics problems appears to be a different concept from the energy concepts discussed (using different symbol conventions) in thermodynamics.”
2. Introduce pseudowork or other work-like quantities along with energy-like equations such as multiple versions of a “work-energy theorem.” While this approach allows problems involving deformable or rotating systems to be solved, it comes at the expense of student conceptual understanding. In particular, introductory physics students will

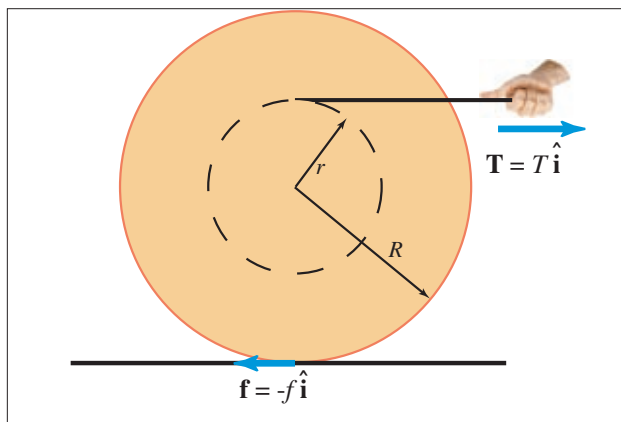


Fig. 1. A spool of radius R is pulled by means of a force \mathbf{T} of magnitude T applied to a string wrapped around an axle of radius r . The spool is in contact with a horizontal table that applies a friction force \mathbf{f} of magnitude f . The spool rolls without slipping.

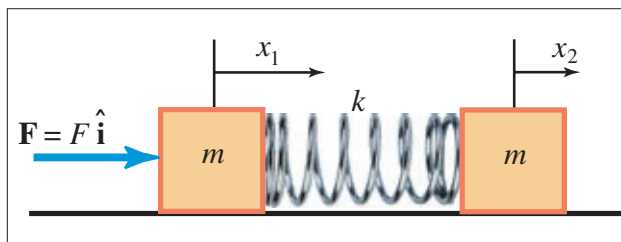


Fig. 2. Two blocks of mass m are connected by a spring of force constant k . The system sits at rest with the spring relaxed on a frictionless table. A constant force \mathbf{F} of magnitude F is applied to the left-hand block, moving it through a distance x_1 . During this time interval, the spring causes the right-hand block to move to the right through a distance x_2 .

have difficulty understanding what is real work and what is pseudowork and what is a real energy equation and what looks very similar, but is not an energy equation.

In this series of articles, an alternative approach is discussed that allows students to solve problems involving deformable or rotating systems in mechanics with well-specified definitions and to see that thermodynamics is consistent with mechanics.

As examples of cases in which the displacement of the center of mass of the object differs from the displacement of the point of application of the force, consider two situations suggested by Mungan.¹² In Fig. 1, a hand pulls with a force of magnitude T on a string wrapped around the axle of a spool. The

spool starts from rest on a table with friction and rolls without slipping. The axle has radius r and the spool has radius R . The center of mass of the spool moves through a horizontal displacement of magnitude L . It is easy to show, however, that the point of application of the force at the hand moves through a displacement with magnitude $L(1 + r/R)$. Therefore, the work done by the hand on the spool-string system is *not* TL , but rather $TL(1 + r/R)$.

The second situation is shown in Fig. 2. A constant horizontal force with magnitude F pushes on a block with mass m , moving it through a distance x_1 . The block is attached to a second identical block by means of a spring of force constant k . While the first block moves through a distance x_1 , the second block moves to the right through a distance x_2 . During this time interval, the center of mass of the system moves through a displacement of magnitude $1/2(x_1 + x_2)$. The work done on the system by the applied force is *not* $1/2Fx_1$, however. The work done on the system by the applied force is Fx_1 because the point of application of the force moves through a displacement¹⁴ of magnitude x_1 .

Net Work

It often occurs that there are multiple forces acting on a system. In this case, it is possible to calculate the net work done by the forces on the system. Unfortunately, a limited definition of the net work is often provided by instructors and authors as if it were generally valid:

The net work done by multiple forces on an object is equal to the product of the net force on the object and the displacement of the object.

In many cases, as indicated in the statement, work is discussed in terms of its effect on an object. One of the emphases in this series of articles will be that it is more fruitful to think about *systems* rather than about *objects*, as discussed in detail in the second article¹⁵ in this series. While a system could indeed be a single object, an emphasis on systems will allow student understanding of a wider variety of problems. The quoted statement above is only true if the system is perfectly rigid and nondeformable. If the system is deformable,

different forces on the system may act through different displacements. Therefore, the statement that provides the correct work done by the forces on the system is:

The net work done by multiple forces on a system is equal to the sum of the works done on the system by each individual force.

The work done by each individual force must be calculated in terms of the displacement of the point of application of the individual force. This latter statement is generally true and is equivalent to the previous statement in the special case of a nondeformable system. Rather than add the forces and then calculate the work, the general approach is to calculate the individual works and then add them together.

The Situation with Friction

Another area in mechanics in which difficulties arise is that of frictional work. If $\Delta \mathbf{r}$ in the definition of work is identified as “the displacement of the object,” it often follows in textbook and lecture discussions that the work done by friction on a block sliding on a surface is $W = -f_k d$, where f_k is the force of kinetic friction on the block and d is the distance through which the block moves relative to the surface. The negative sign indicates that the friction force is in the opposite direction to the displacement. This expression for work is then incorporated into the work-kinetic energy theorem for the block.

This approach ignores the fact that the displacement of the block is not the same as the many displacements of the friction force at a large number of contact points. This latter displacement is complicated and involves deformations of the lower surface of the block. This issue has been discussed in the literature.^{16,17}

A student in a calculus-based class may have little difficulty with $W = -f_k d$, based on his or her understanding of evaluating the work done by any force by performing a path integral over the path followed by the object. In the case of a block sliding over a stationary surface, the friction force is always oppositely directed to each infinitesimal displacement of the block. For a constant friction force, this integral reduces to the product of the force and the length of the path.

What about the student in the noncalculus class, however? This student does not see the path integral formulation of the work. The bright student in such a class will consider $W = -f_k d$ and wonder why work is now defined in terms of a *distance* rather than a *displacement*. Such an equation creates a jarring disconnect with earlier discussions of work in which the work was calculated by means of a displacement. Furthermore, the noncalculus student may give no thought to infinitesimal displacements and may generalize $\Delta \mathbf{r}$ in the definition of work to mean only macroscopic displacements. For an object moving on a curved path, d is the length of the path, which could be quite different from the magnitude of the displacement of the object.

In my opinion, a more conceptually fruitful approach for a situation such as a block sliding on a surface is to (1) drop the phrase “work done by friction,” (2) *not* invoke the work-kinetic energy theorem, and (3) identify the combination $-f_k d$ with the change of mechanical energy E_{mech} of the system involving the block and the surface with which it is in contact:¹⁸

$$-f_k d = \Delta E_{\text{mech}}. \quad (2)$$

Equation (2) represents only that part of the change in mechanical energy of the system caused by friction forces. Of course, if other forces act, such as a hand pushing the block or gravity pulling the block down an inclined surface, the mechanical energy of the system will also change due to these forces. In a case such as the block sliding down an inclined surface, the system could be expanded to include the Earth so that the mechanical energy of the system would include both kinetic energy and gravitational potential energy.

Regardless of whether other forces besides friction act on the block, the decrease in mechanical energy in Eq. (2) corresponds to an increase in internal energy in the system:

$$+f_k d = \Delta E_{\text{int}}, \quad (3)$$

where the internal energy is shared between the block and the surface. While this approach results in the same mathematical steps in energy problems involving friction as the approach involving $W =$

$-f_k d$, it removes the conceptual difficulties and inconsistencies for the student.

One important point that should not be overlooked is suggested by the discussion above. In the energy approach discussed in this series of articles, the *work-kinetic energy theorem is not a starting point for problem-solving*. While the work-kinetic energy theorem may be included in some problem solutions, it is not used by students as a fundamental principle to begin an energy problem. This important difference from more traditional approaches is discussed more fully in the fourth article¹⁹ in this series.

Questions

In light of these discussions, consider the following true-false questions:

- (1) **True or False?** A boy jumps up into the air by applying a force downward on the ground. The work-kinetic energy theorem $W = \Delta K$ can be applied to the boy to find the speed with which he leaves the ground.
- (2) **True or False?** A balloon is compressed uniformly from all sides. Because there is no displacement of the balloon's center of mass, no work is done on the balloon.

Both of these claims are false. Question (1) refers to a simple, everyday experience that unfortunately cannot be analyzed by means of traditional physics teaching without the introduction of additional work-like quantities and energy-like equations. The upward force on the boy that projects him into the air is the normal force on his feet from the ground. The center of mass of the boy indeed moves through an upward displacement. The normal force, however, goes through no displacement in the reference frame of the ground, and therefore no work is done by this force on the boy. The change in the boy's kinetic energy does not come from work done on the system of the boy. This is a case of a deformable system. Other cases include a person climbing stairs or a ladder, a girl pushing off a wall while standing on a skateboard, and a piece of putty slamming into a wall. In all of these cases, no work is done by the contact force, because there is no displacement of the point of application of the force in the inertial frame

of the surface applying the force.²⁰ These are common situations, however, and our physics instruction should allow students to analyze them correctly. With the correct definition of work and a more global approach to energy,¹⁹ the student can correctly identify these situations as involving conversions of energy within the system rather than as applications of the work-kinetic energy theorem.

Question (2) directly attacks the definition of work when the student arrives at thermodynamics. If $\Delta \mathbf{r}$ has been defined as “the displacement of the object,” it would appear that no work is done on the balloon, because it experiences no displacement. If, however, $\Delta \mathbf{r}$ has been defined as “the displacement of the point of application of the force,” it is clear that the forces compressing the balloon at its inwardly moving surface have moved through displacements so that work has indeed been done by these forces on the balloon. In this case, the work causes an increase in the internal energy of the gas in the balloon.

Question (2) also relates to the discussion in the section on net work. The net force on the balloon is zero because it remains stationary. An approach that evaluates the net work from the net force would again result in zero work done on the balloon, which we know to be incorrect.

Conclusion

Much can be done to clarify the definition of work in textbook and classroom presentations to avoid conceptual inconsistencies and to remove the need for later correction to earlier statements and definitions. It is possible to solve problems in physics without the need for introducing additional work-like quantities and energy-like equations. Not only will a simpler approach help students to understand more sophisticated problems in mechanics, but it will help them make the important connection between mechanics and thermodynamics. In the next installment of this series, we will discuss confusion generated by the failure to identify the correct system when discussing energy.

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12. C.E. Mungan, “A primer on work-energy relationships for introductory physics,” *Phys. Teach.* **43**, 10–16 (Jan. 2005).
13. As a textbook author myself, I do not specifically identify problematic statements in other authors’ textbooks in this series of articles. I do not want this series to appear as a marketing tool but rather as a professional communication that offers a set of suggestions for improving the teaching of energy to our students. I present items from several textbooks in general terms and not as direct quotes.
14. Full solutions to these two problems appear in J.W. Jewett, “Energy and the confused student V: The energy/momentum approach to problems involving rotating and deformable systems,” *Phys. Teach.*, to be published in May 2008.
15. J.W. Jewett, “Energy and the confused student II: Systems,” *Phys. Teach.*, to be published in Feb. 2008.
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18. See, for example, R.A. Serway and J.W. Jewett, *Physics for Scientists and Engineers*, 7th ed. (Brooks/Cole, Belmont CA, 2008), pp. 204–209 or R.A. Serway and J.W. Jewett, *Principles of Physics*, 4th ed. (Brooks/Cole, Belmont CA, 2006), pp. 173–177.
19. J.W. Jewett, “Energy and the confused student IV: A global approach to energy,” *Phys. Teach.*, to be published in April 2008.
20. This statement assumes that the surface applying the force is nondeformable. In reality, the surface will deform slightly; the ground depresses downward a bit when the boy jumps upward, the stairs or ladder rungs deform a small amount downward when the person climbs upward, the wall bends inward slightly when the girl pushes off or the putty slams into it. In these cases, the direction of the displacement of the point of application of the force is opposite that of the normal force exerted by the surface, so the work done on the boy, person, girl, or putty by the surface has a small negative value.

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